Earth Satellites
Motion in the gravitational field

dr inż. Stefan Jankowski
s.jankowski@am.szczecin.pl

Satellites

- There are two groups of satellites: natural (planets for the Sun, moons for planets) and artificial ones - developed and launched by human
- An artificial satellite is any object, which at a certain height above the surface of the Earth has a sufficient speed to move on an Earth orbit, that can be circular or elliptical

The forces acting on the satellite

- Stationary forces:
  determine the stationarity of the movement process, are easily predictable, favorably affect the stabilization of the trajectory
  - The Earth gravity; an idealized movement according to Kepler laws
- Perturbative forces (disturbances):
  disturb the stationarity of the satellite motion process; often difficult to estimate
  - related to an eccentricity of the gravitational field
  - others, not related to the gravity

Disturbances – factors which influence on gravity

- Heterogeneity of Earth masses
- Flattening of the Earth
- The asymmetry of Earth masses relative to the equator
- ellipticity of the equator
- Tides of the seas and land areas
- Gravity of other celestial bodies, mostly the Sun and the Moon
- Electromagnetic forces

Disturbances –
factors which are not related to the gravity

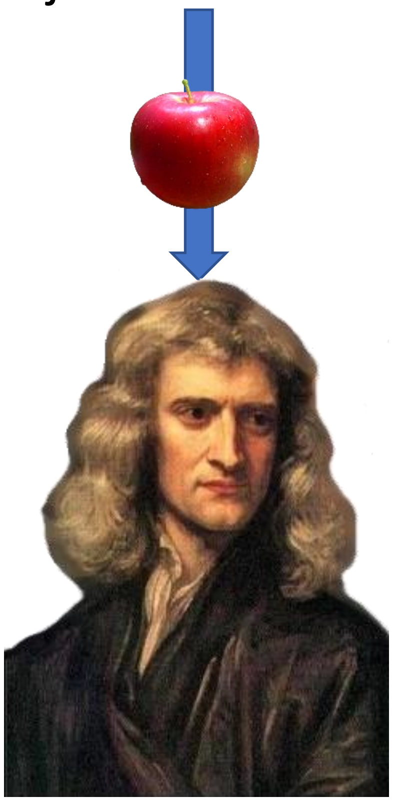
- resistance of the atmosphere
- the pressure of the sunlight (non-continuous character - only when the satellite is lit by the Sun),
- other factors: radiation reflected from the Earth, cosmic dust, relativistic effect, etc.

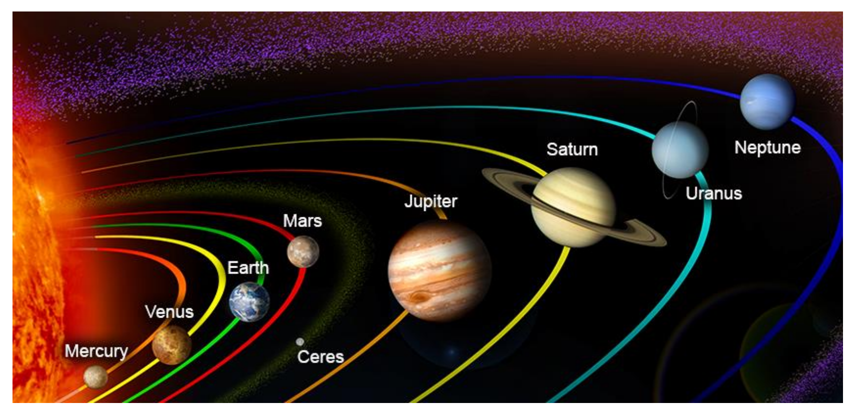
Kepler’s laws – idealized movements

- Developed by Jan Kepler (1571 – 1630) basing on Mars movement around the Sun
- He used observations made by Tycho Brahe, who even wasn't convinced to heliocentric theory worked out by Mikolay Copernicus

The Gravity

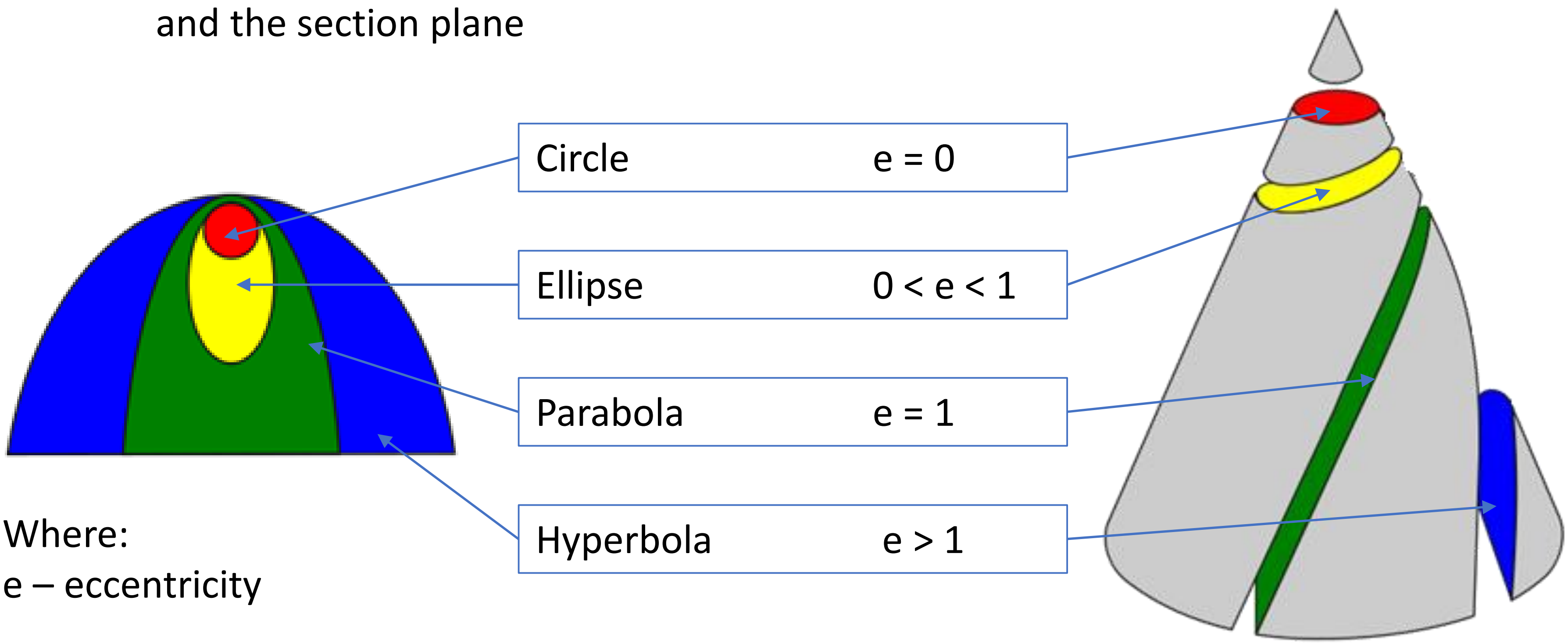
- Isaac Newton (1643-1727) showed that Kepler’s laws relate to all objects with mass and result from the law of universal gravitation





I Kepler's law

- The satellite orbit is a conic section (ellipse, parabola or hyperbola) with the earth's gravity center in the one of its focuses
  - a conic section (or simply conic) is a curve obtained as the intersection of the surface of a cone with a plane
  - the shape of the conic section depends on the angle between the cone axis and the section plane



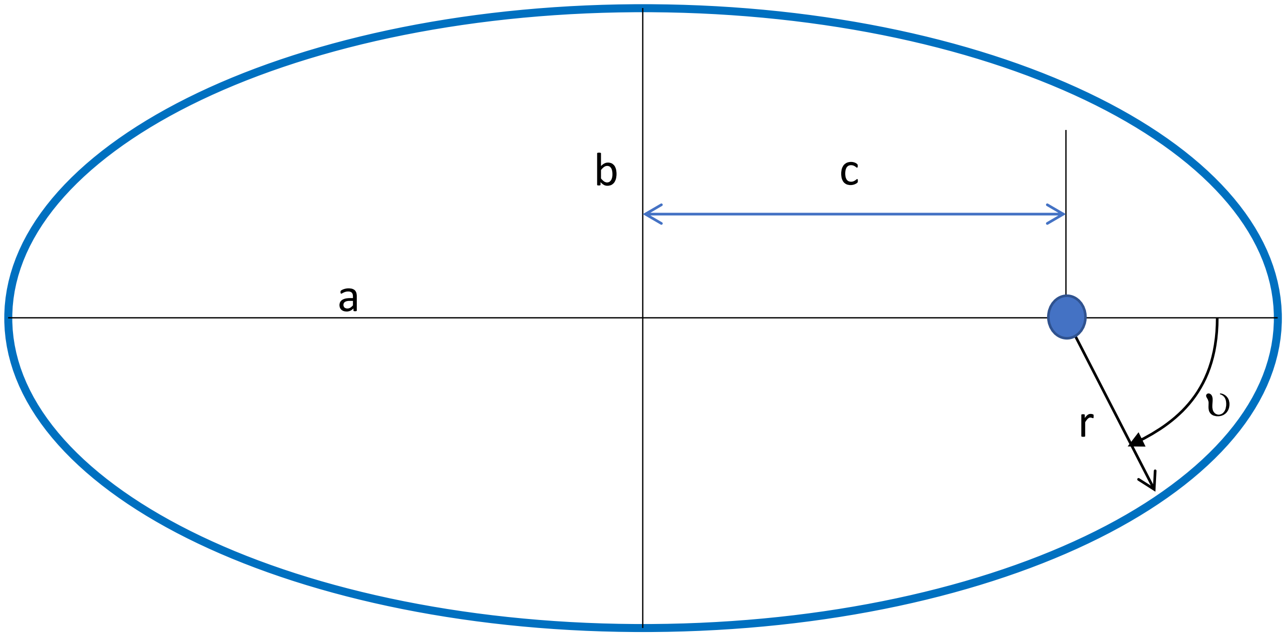
Where:
e – eccentricity

The orbit's shape – an ellipse

$$\mathrm{r} = \frac{b}{1 + e \cos v}$$

$$b = a(1 - e^2)$$

$$\mathrm{e} = \frac{c}{a}$$



where:

a	–	semi-major axis
b	–	semi-minor axis
υ	–	angle of a satellite's position vector from the perigee (true anomaly)
e	–	eccentricity
c	–	focal distance
r	–	a satellite's position vector; a distance between the earth's gravity center and the satellite

The orbital shape vs the linear velocity of a satellite

- The shape of the orbit depends on the satellite's speed
- The minimum speed of the satellite required on a specific orbit can be found as follows:

$$F_d = F_g$$

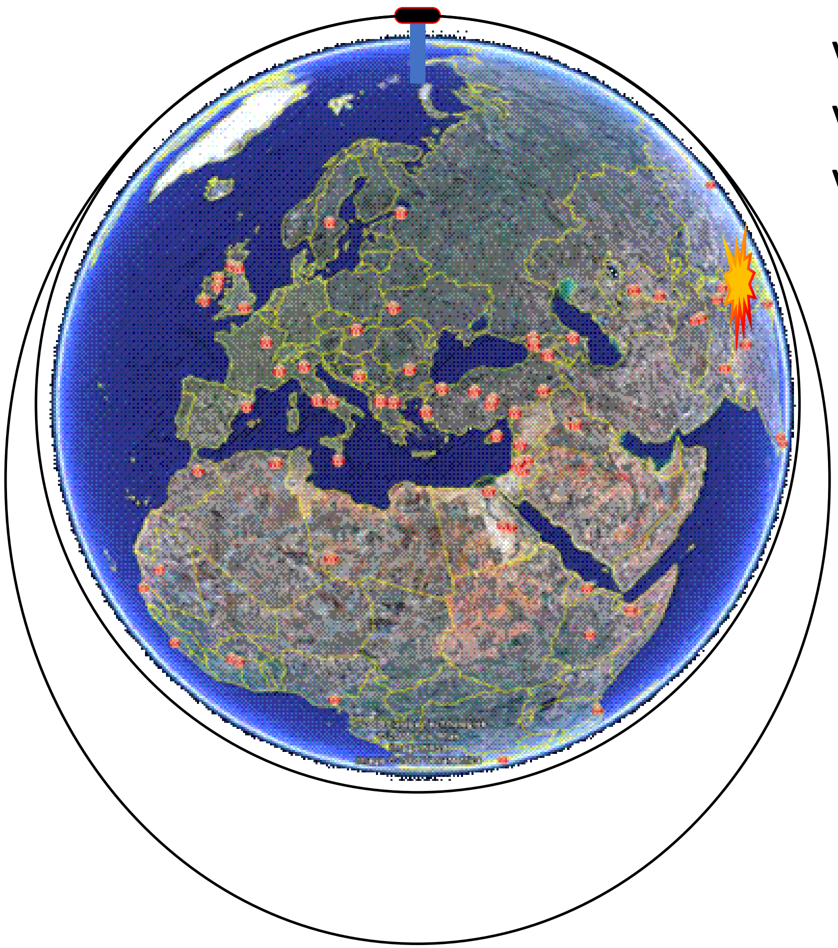
$$\frac{mV_I^2}{r} = G\frac{Mm}{r^2}$$

$$V_I = \sqrt{\frac{GM}{r}}$$

where:
F_d – centripetal force [N]
F_g – gravity [N]
M – mass of the Earth $5.976 \cdot 10^{24}$ [kg]
G – general gravitational constans $6.67 \cdot 10^{-11}$ $m^3/(kg \cdot s^2)$
m – mass of the satellite
V – linear speed
r – distance between the Earth's gravity center and a satellite's gravity center; radius of the orbit [m]

For r = $6.37 \cdot 10^6$ [m] – mean Earth's radius,
$V_I \approx 7.91$ [km/s] **The first cosmic velocity**

The orbital shape:



$V < V_I$	elliptical?!
$V = V_I$	circular
$V_I < V < V_{II}$	elliptical
$V = V_{II}$	parabolic
$V > V_{II}$	hyperbolic

The second cosmic velocity

- the minimum speed of satellite, which allowed it to recede from the Earth into infinity, on parabolic or hyperbolic orbit
- It can be calculated by comparing the energy of the object located on the surface of the Earth (a celestial body) and in infinity. The energy in infinity is equal to 0 (both kinetic energy and potential energy of the gravitational field), therefore:

$$\frac{mV_{II}^2}{2} - G\frac{Mm}{r} = 0$$

$$V_{II} = \sqrt{\frac{2GM}{r}} = \sqrt{2}V_I$$

For $r = 6.37 \cdot 10^6$ [m] – mean Earth's radius,

$V_{II} \approx 11.2$ [km/s] The second cosmic velocity

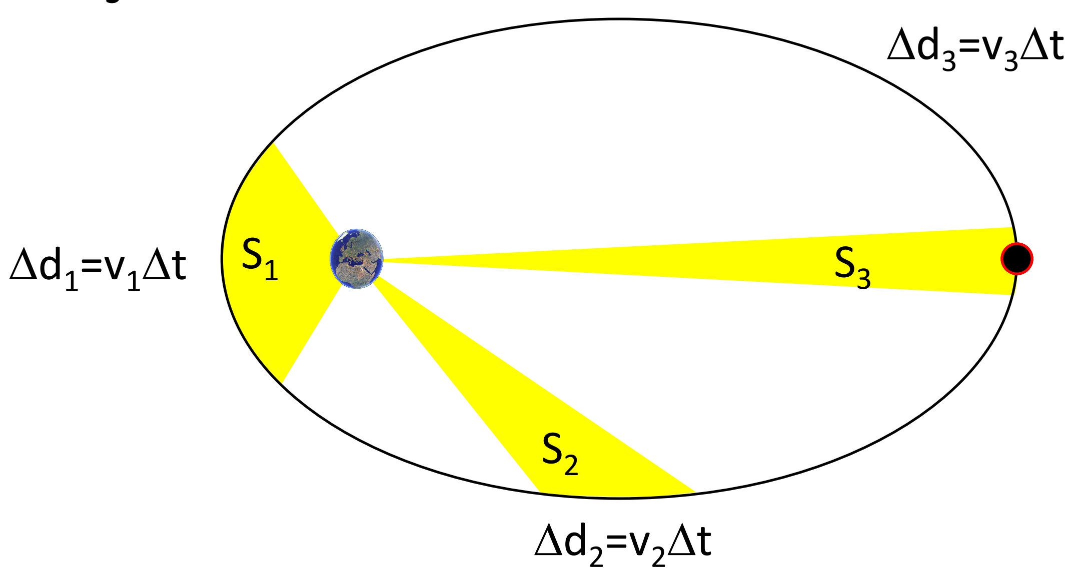
II Kepler's law

- The satellite's position vector (from earth's gravity center to the satellite) sweeps over the same areas within the same time interval
    - The linear velocity of the satellite on an elliptical orbit is not constant. In the perigee, the satellite moves faster than in the apogee
    - Within the same time **Δt**, the satellite travels longer distance **Δd_1** near perigee than near the apogee **Δd_3**.

$$S_1 = S_2 = S_3$$

$$r^2 \frac{dv_i}{dt} = S_i = const$$

$$\Delta d_1 > \Delta d_2 > \Delta d_3$$

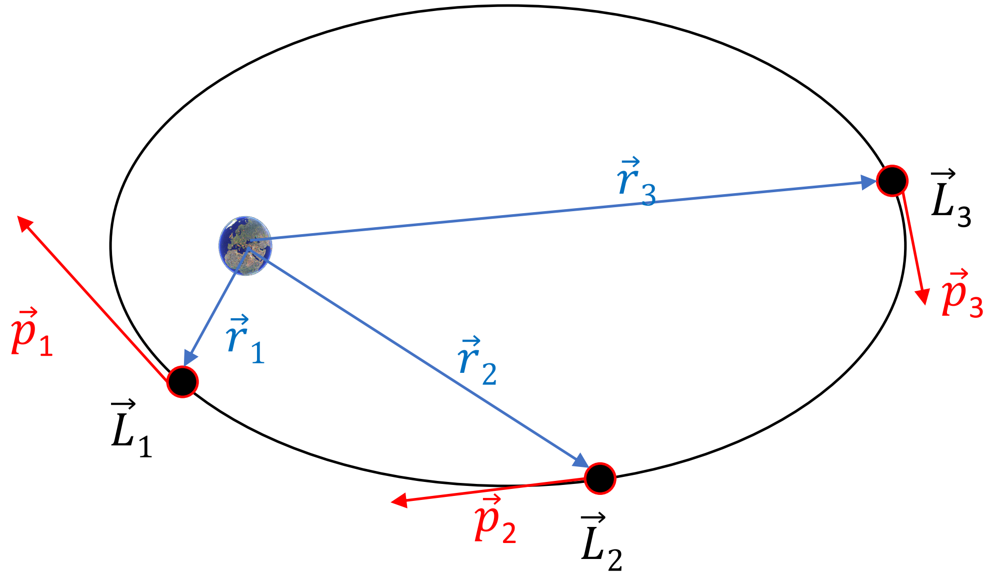


The law of conservation of angular momentum (II KL)

- The II Kepler's law is an equivalent of the law of conservation of angular momentum:

$$\vec{L} = \vec{r} \times \vec{p} = const$$

$$\vec{p} = m\vec{v}$$



gdzie:

$\vec{L}$ – an angular momentum

$\vec{r}$ – a position vector

$\vec{p}$ – a momentum

$$\overrightarrow{L_1} = \overrightarrow{L_2} = \overrightarrow{L_3} = const$$

$$\overrightarrow{p_1} > \overrightarrow{p_2} > \overrightarrow{p_3}$$

$$\overrightarrow{r_3} > \overrightarrow{r_2} > \overrightarrow{r_1}$$

III Kepler’s law

- When the satellite travels in an elliptical orbit, the square of the orbital time is proportional with the semi-major axis to the power of three

$$\frac{a_1^3}{T_1^2} = \frac{a_2^3}{T_2^2} = const$$

Where:
a – semi-major axis
T – orbital time

III Kepler's law – Newton's interpretation

For circular orbits:

$$G\frac{Mm}{r^2} = \frac{mV^2}{r}$$

$$V = \frac{2\pi r}{T}$$

$$G\frac{Mm}{r^2} = \frac{m\left(\frac{2\pi r}{T}\right)^2}{r}$$

$$G\frac{Mm}{r^2} = \frac{m4\pi^2 r^2}{r}$$

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2} = const$$

$$\frac{r_1^3}{T_1^2} = \frac{r_2^3}{T_2^2}$$

$$\frac{r_1^3}{r_2^3} = \frac{T_1^2}{T_2^2}$$

General expression for two satellites on elliptical orbits:

$$\frac{T_1^2(M+m_1)}{T_2^2(M+m_2)} = \frac{a_1^3}{a_2^3}$$

where:
T_1, T_2 are orbital times (orbital periods) of satellites with masses m_1, m_2 around central celestial body (e.g. the Earth)

Orbital periods

For Earth’s satellites, the orbital period **T** is counted in hours and minutes or as a part of the sidereal day, as follows:

$$T = \frac{n}{m}$$

Where **m** is a whole number of full revolutions of the satellite around the Earth during the whole number of sidereal days **n**

Celestial coordinates – orbit’s parameters

In order to define a position and movement of a satellite in relation to an observer, the knowledge about coordinate system (called rectascension system) is required. The system describes an orbit's parameters known as Kepler parameters.

There are six parameters:

Ω - rectascension for ascending node

i - inclination

ω - perigee argument

a - semi-major axis

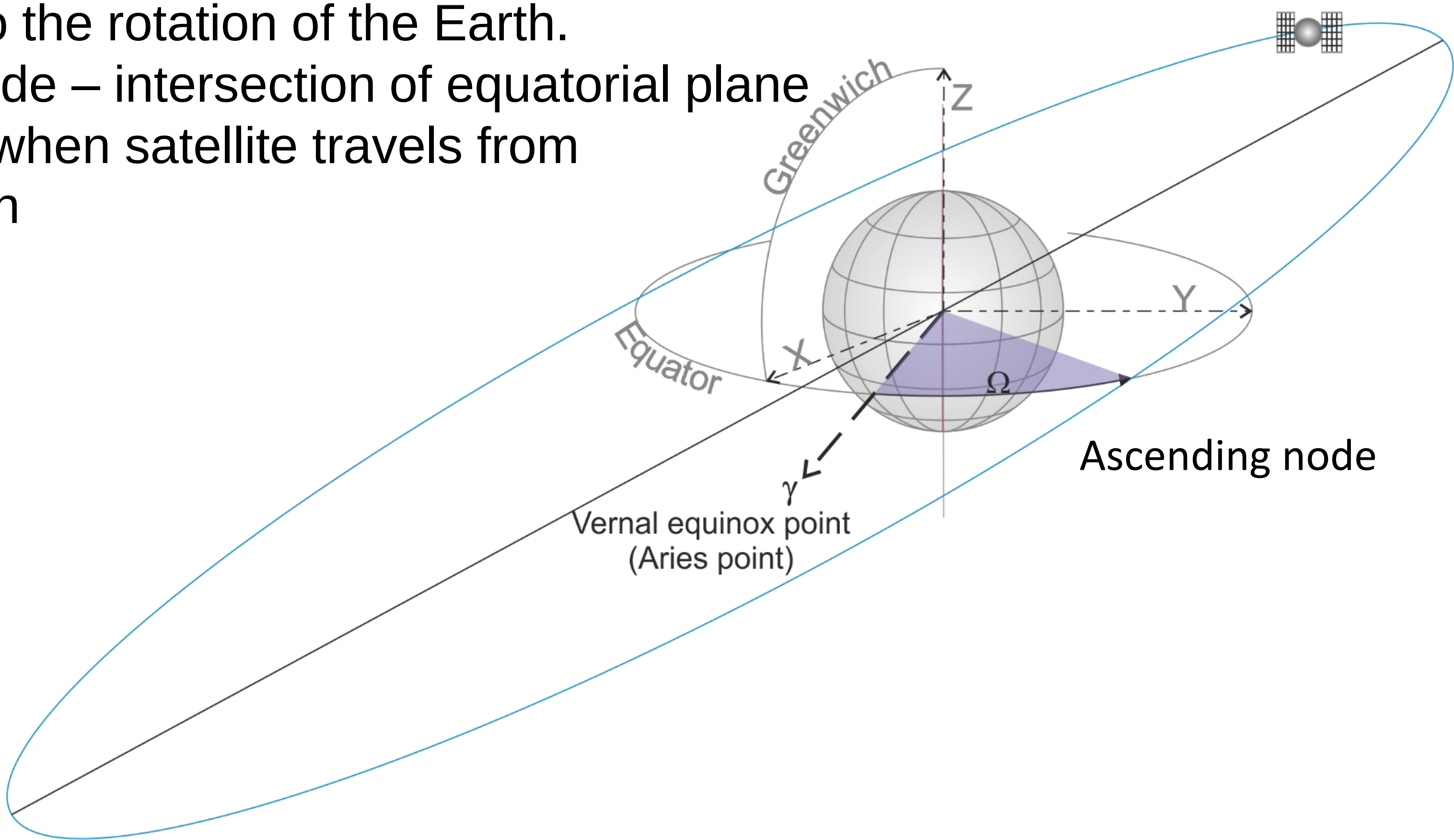
e - the orbit’s eccentricity

υ - the satellite’s true anomaly

The movement of the satellite is connected with a time by specifying the moment when the satellite pass through the selected point of the orbit: e.g. the perigee.

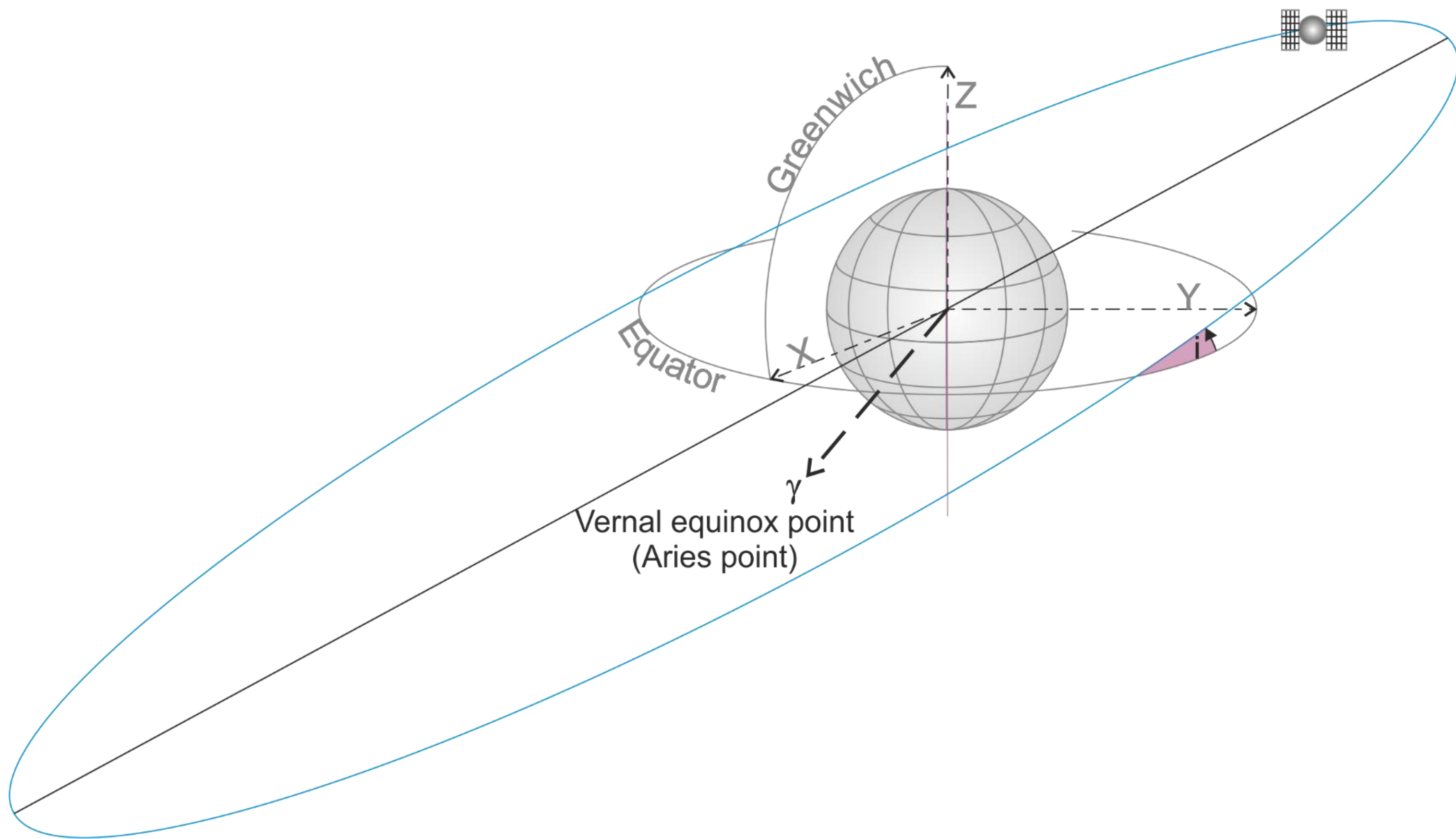
Rectascension - Ω

Right Ascension of Ascending Node (RAAN) – angle measured in the plane of the equator from the reference point on the horizon, which is called the spring equinox (Vernal equinox point) to ascending node.
The reference point is not related to the Earth's coordinate system due to the rotation of the Earth.
Ascending node – intersection of equatorial plane and the orbit when satellite travels from South to North



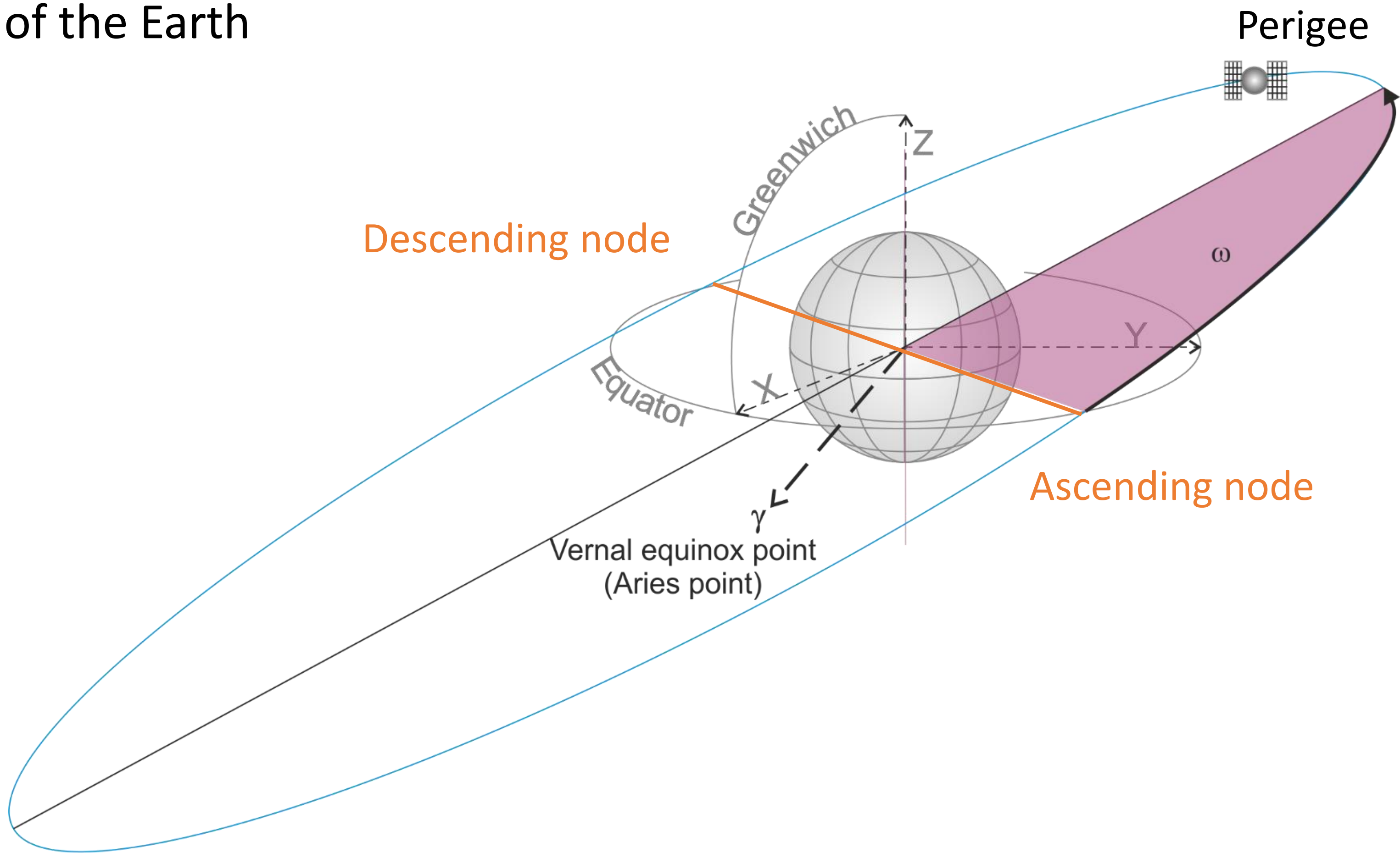
Inclination – i

the angle between the equator plane and the orbit plane



Perigee argument – ω

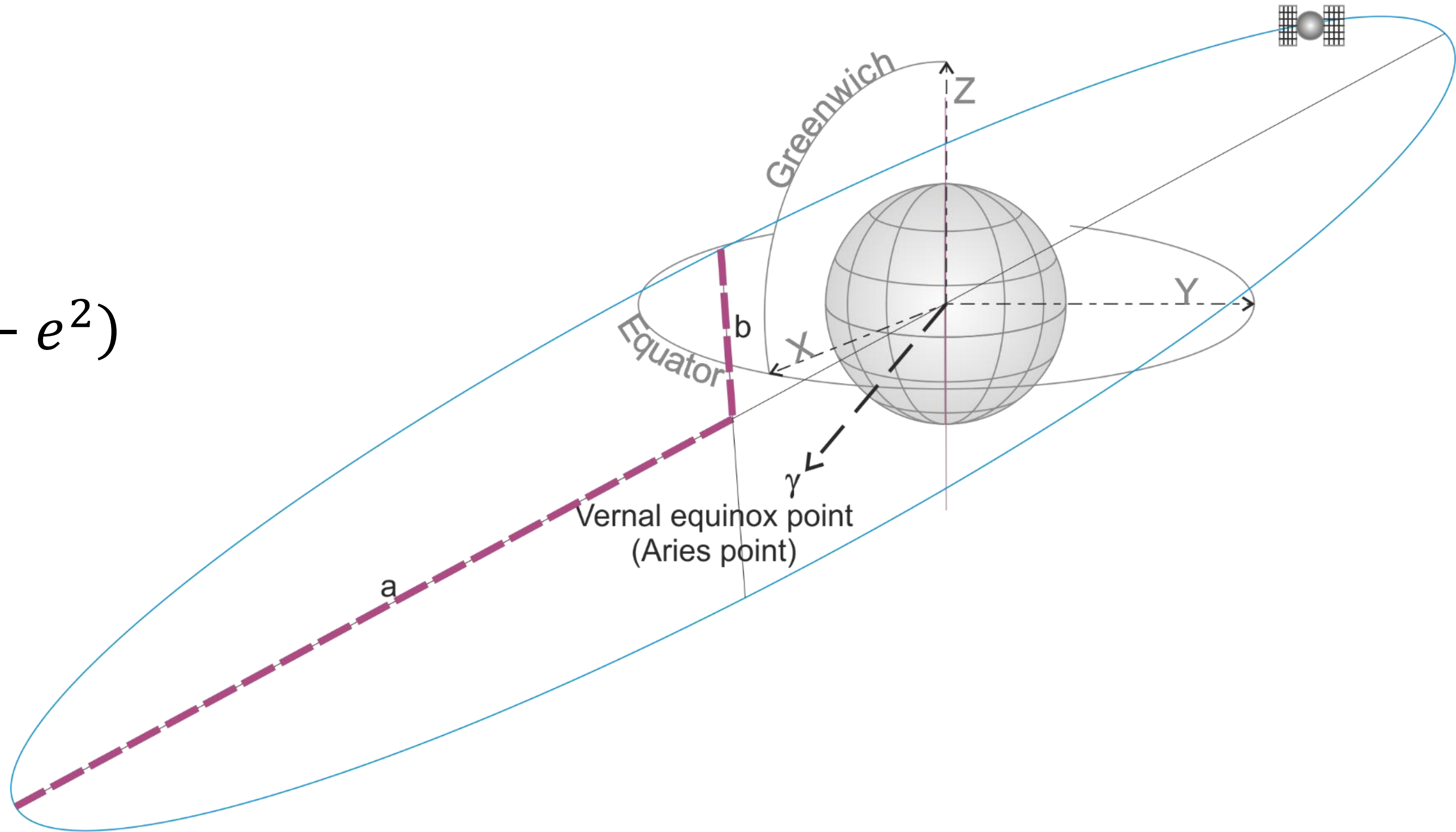
the angle between the nodal line and the line of apsides passing through the perigee, the apogee and the center of the Earth



Semi-major axis – a

a semi-major axis of the elliptical orbit determines its size and determines the period time of the satellite's circulation around the central body

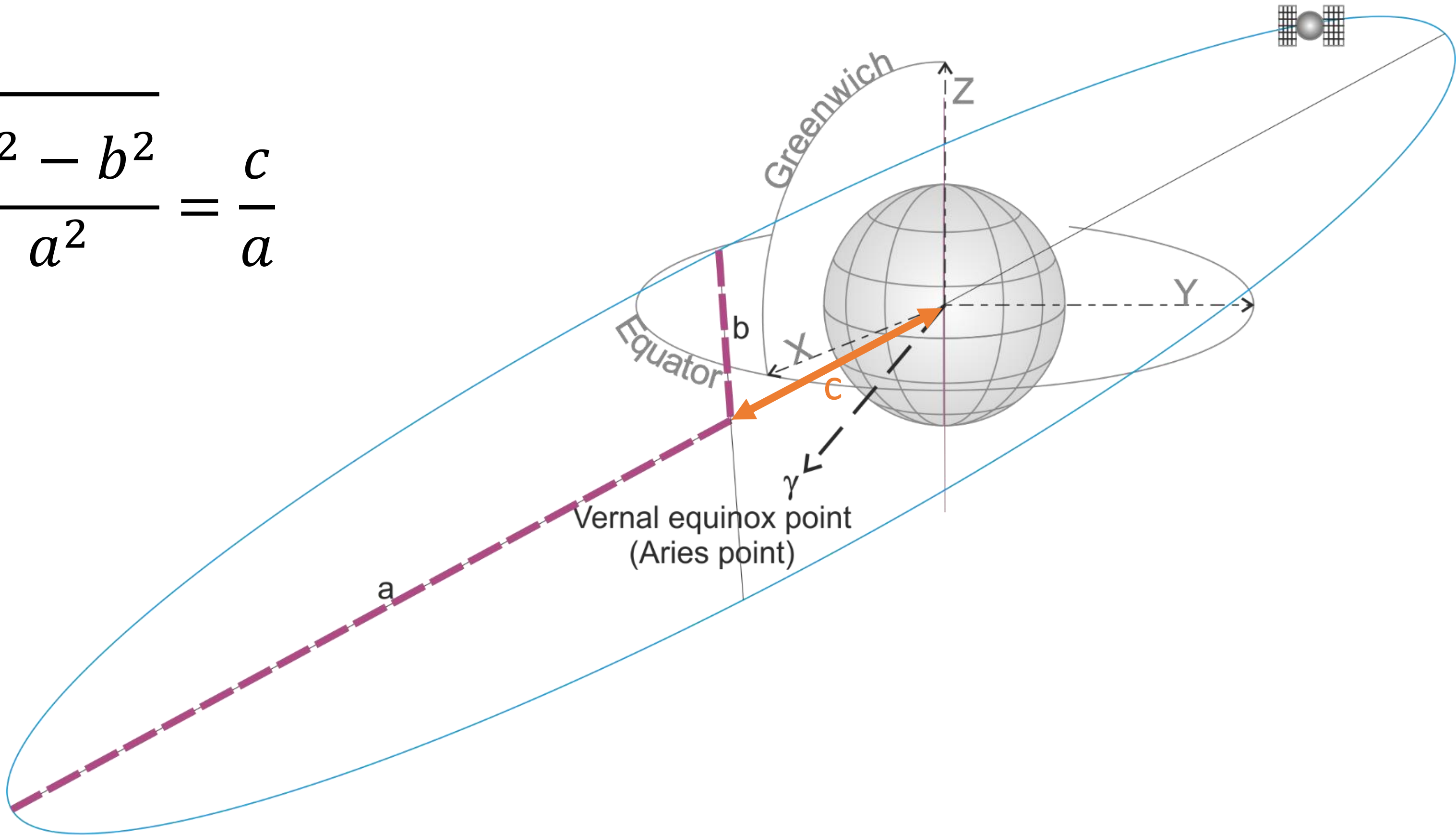
$$b = a(1 - e^2)$$



Eccentricity – e

determines the shape of the ellipse; for e = 0 the ellipse is a circle; when e approaches 1 the ellipse becomes long and flat

$$e = \sqrt{\frac{a^2 - b^2}{a^2}} = \frac{c}{a}$$

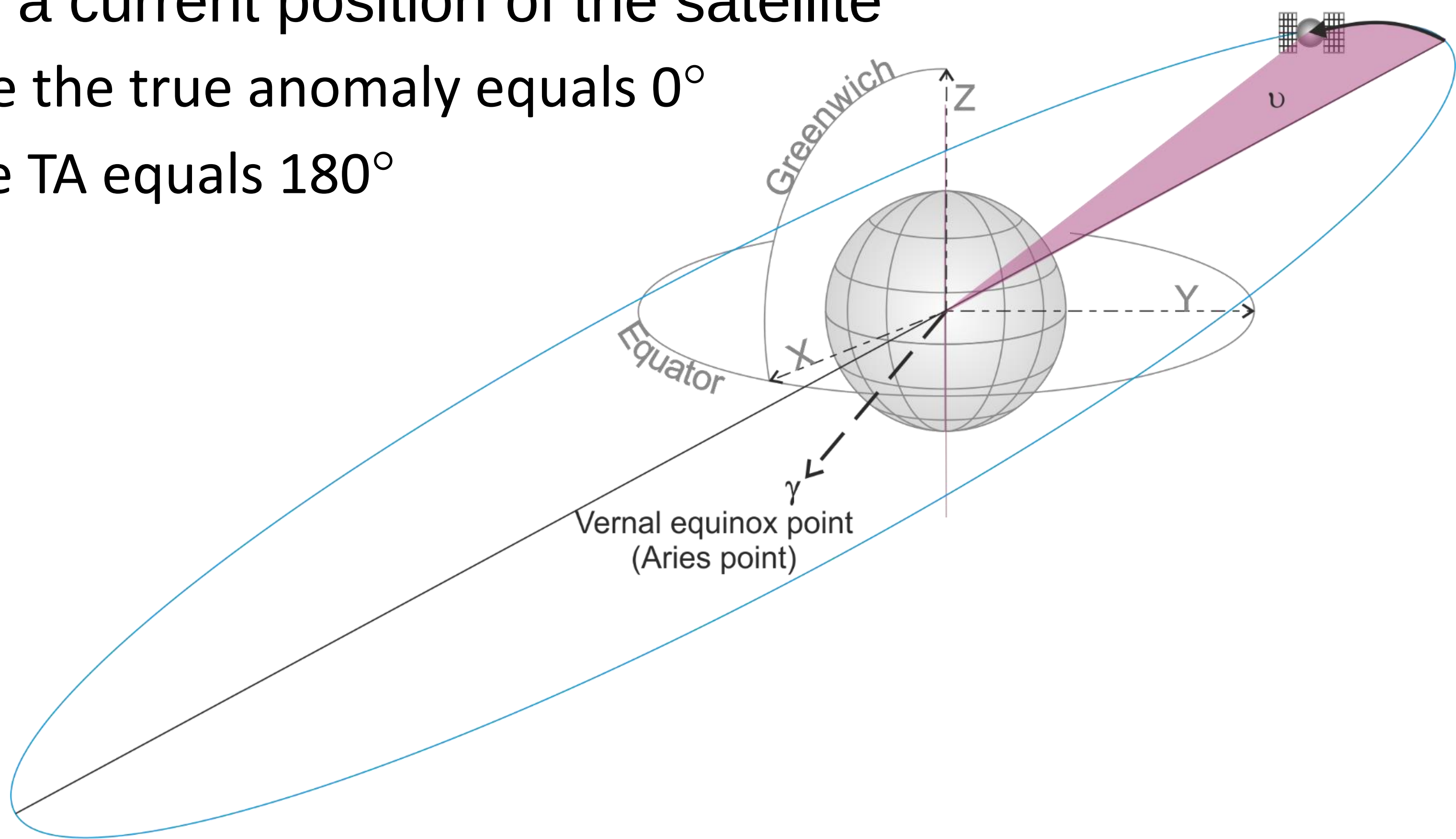


The satellite's true anomaly – υ

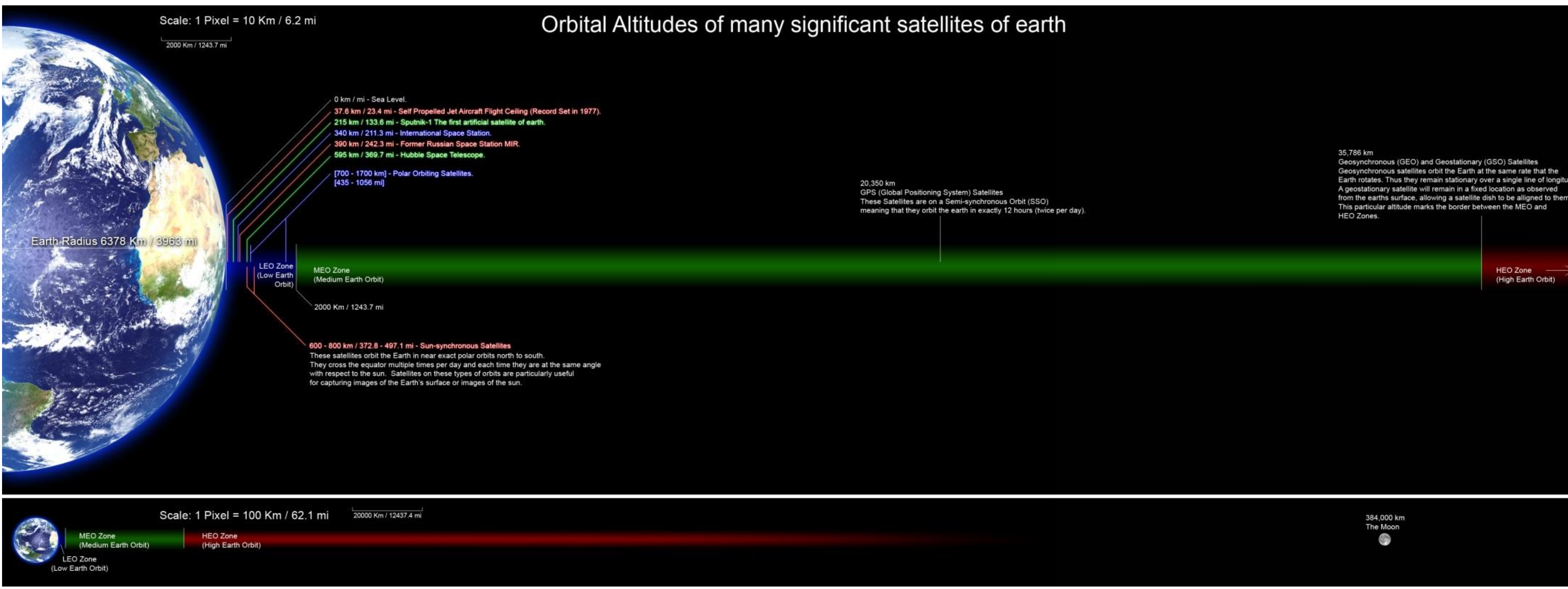
angle measured in the plane of the orbit from the perigee (line of apsides) to a line drawn from the Earth's gravity center to a current position of the satellite

At perigee the true anomaly equals 0°

At apogee TA equals 180°

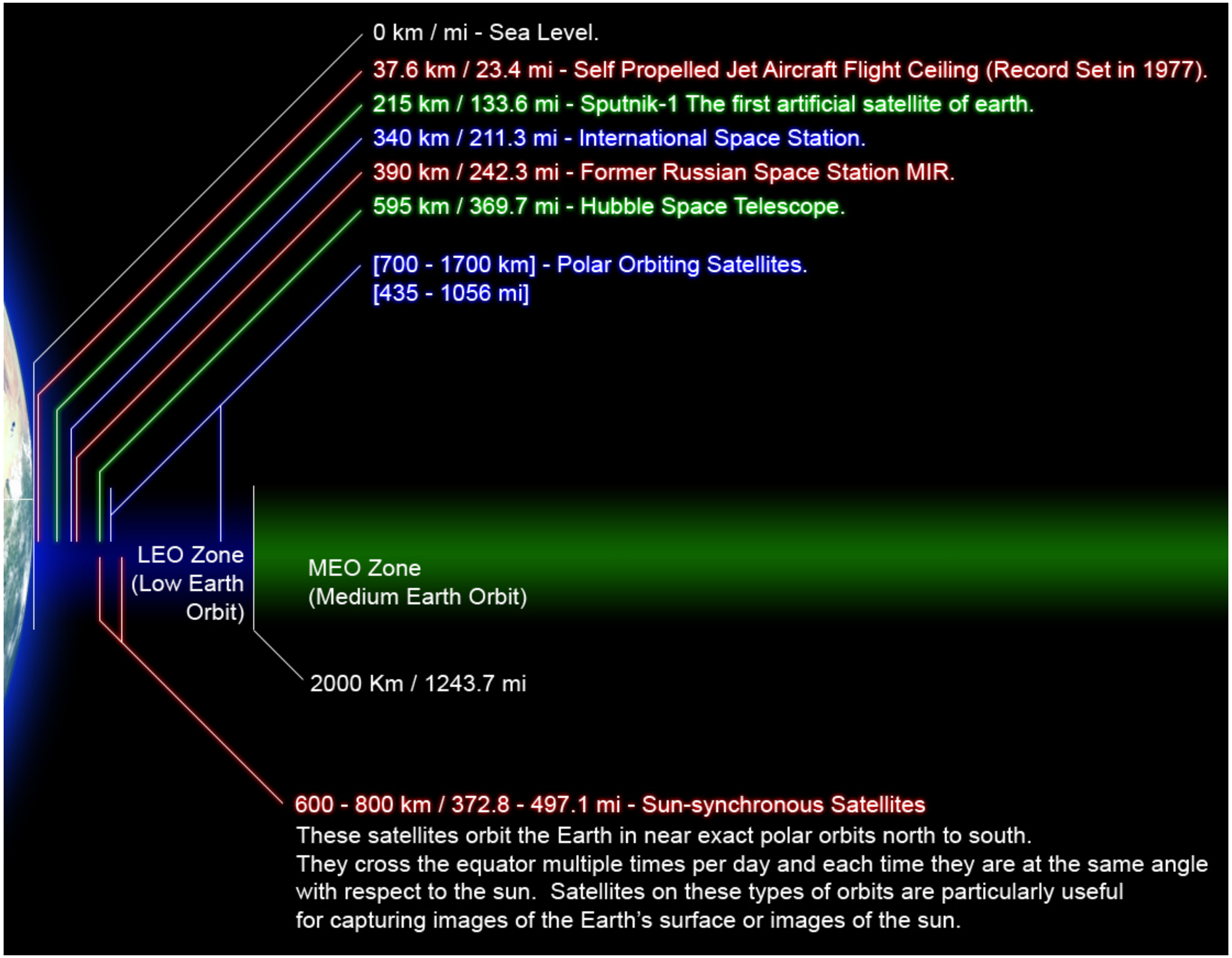


Orbits

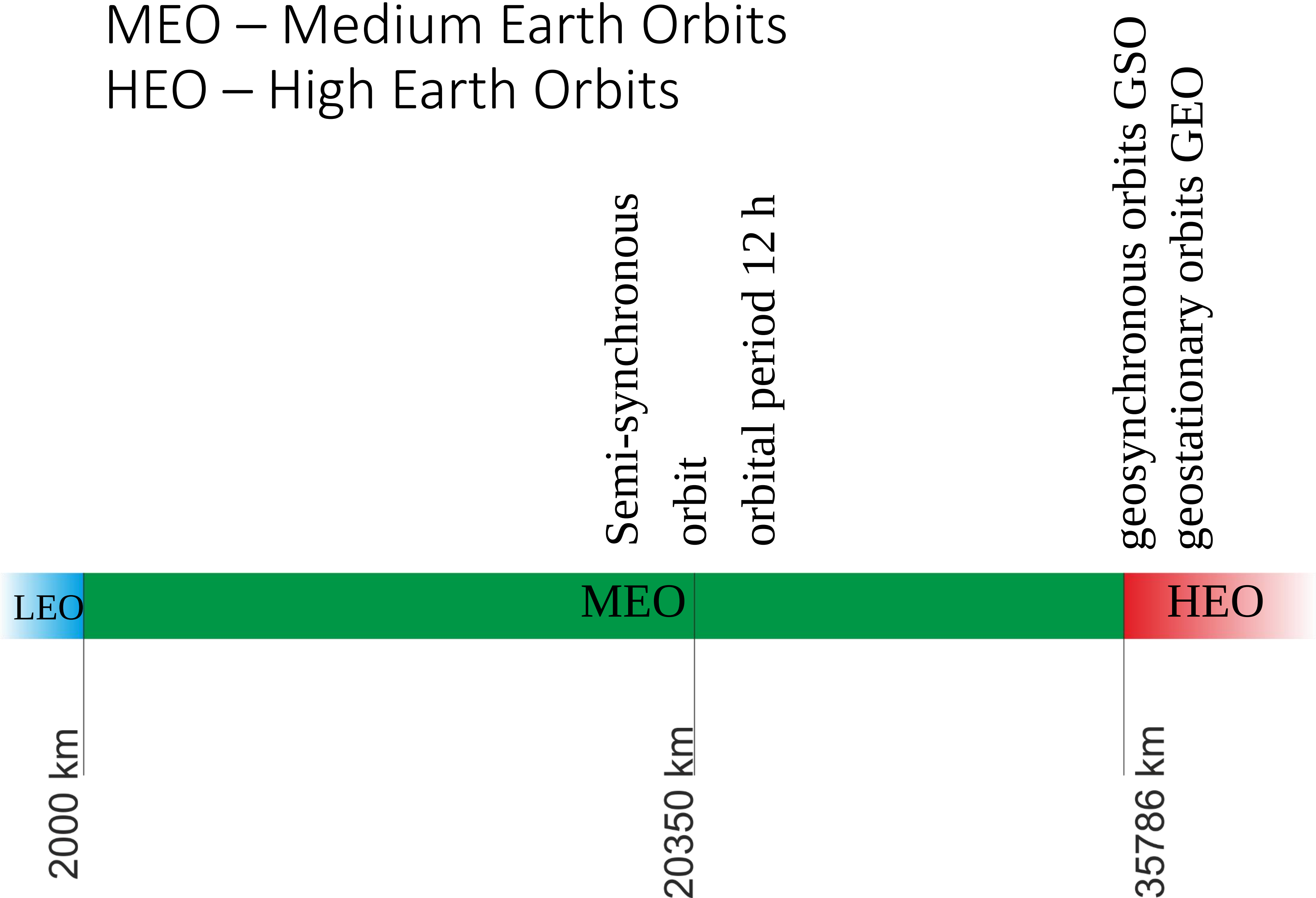


- http://commons.wikimedia.org/wiki/File:Orbitalaltitudes.jpg

LEO – Low Earth Orbits

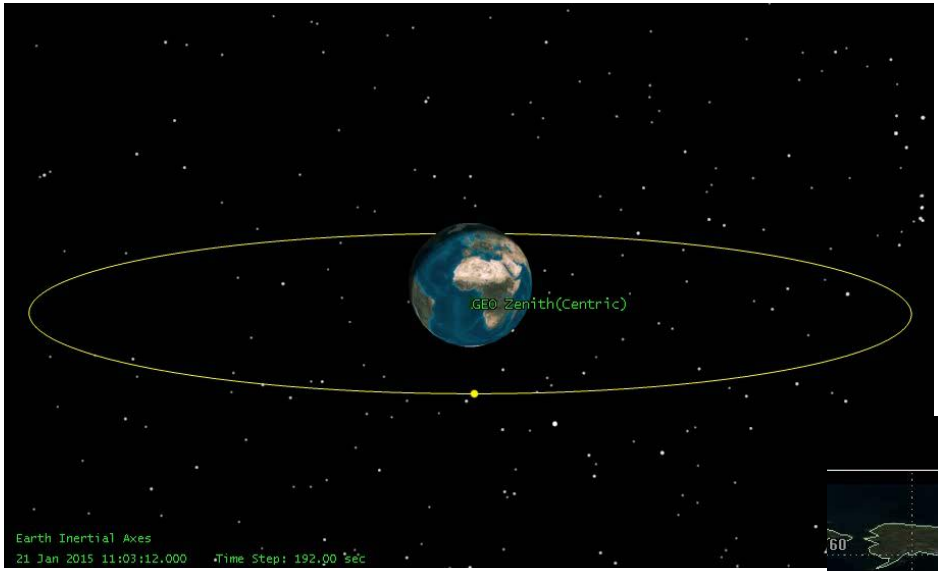


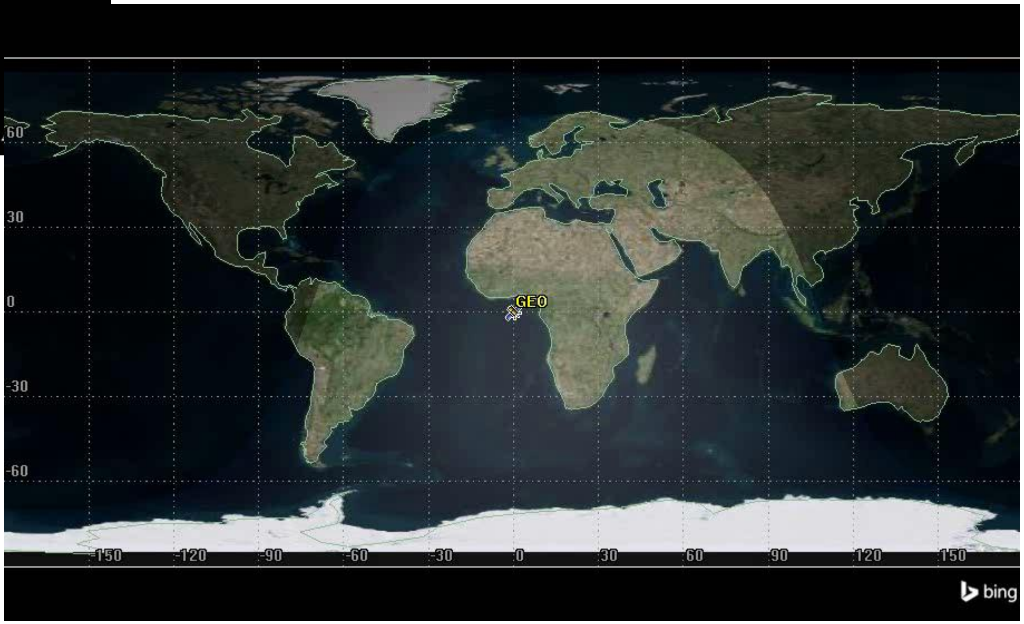
MEO – Medium Earth Orbits
HEO – High Earth Orbits



Geostationary orbit GEO

T = 1
i = 0°
e = 0





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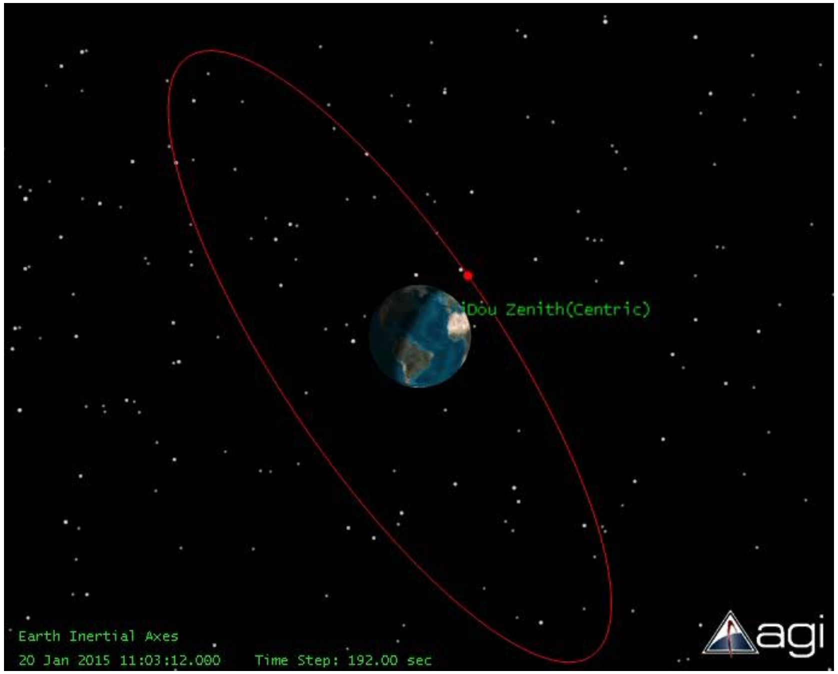
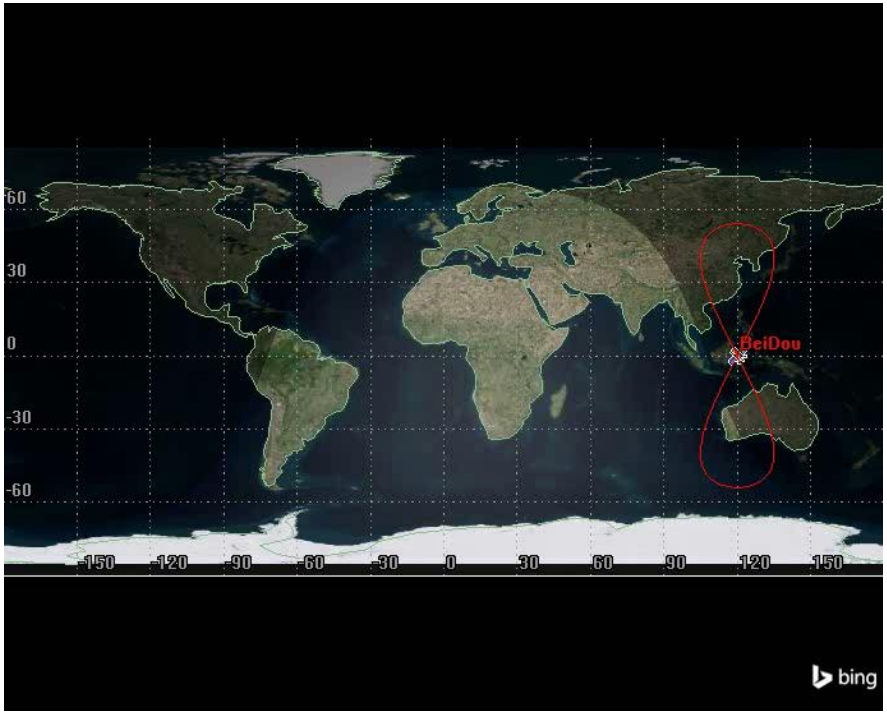
Geosynchronous orbit GSO

T = 1
i = 55°
e = 0





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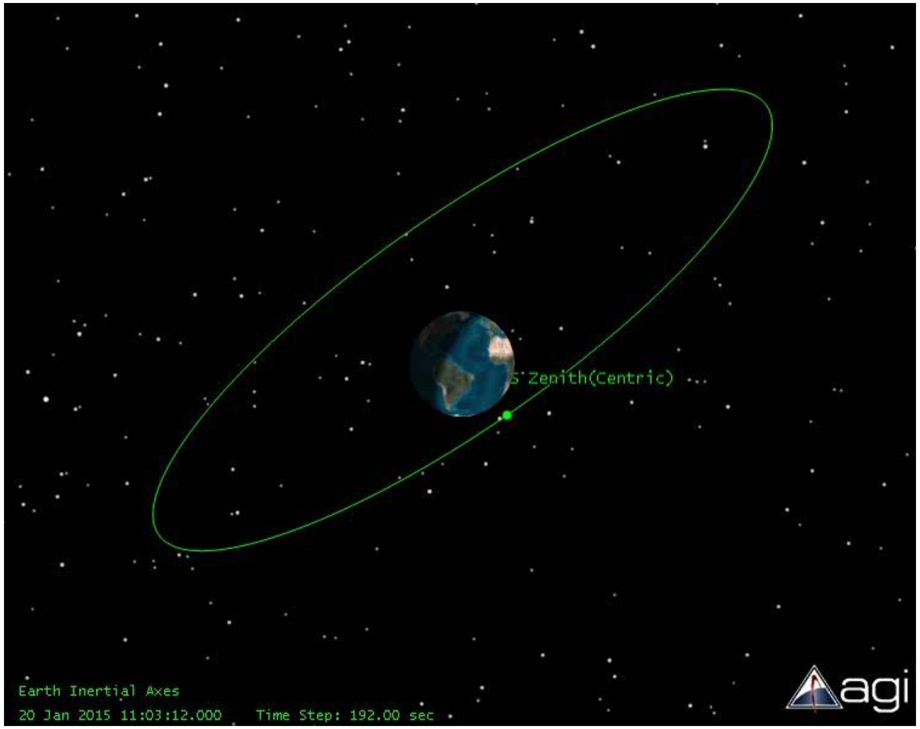
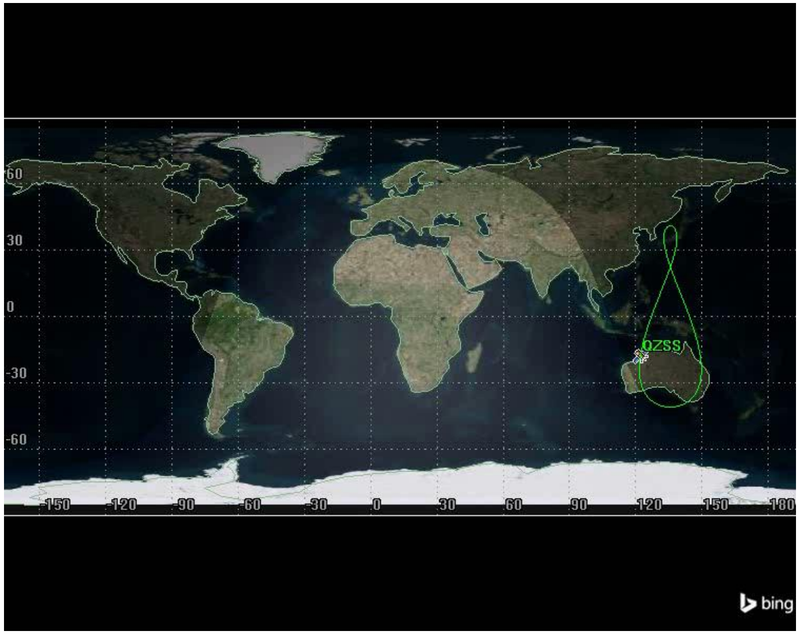
Geosynchronous orbit GSO

T = 1
i = 41°
e = 0.075





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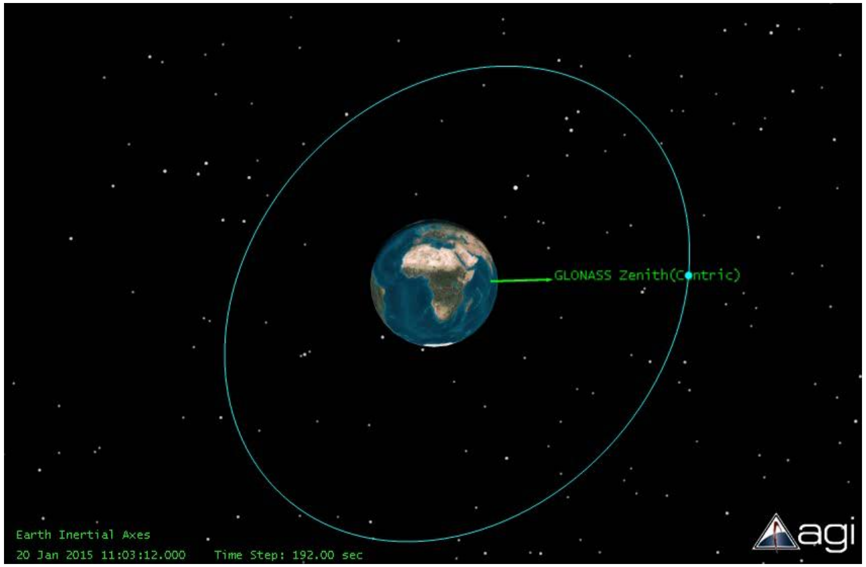
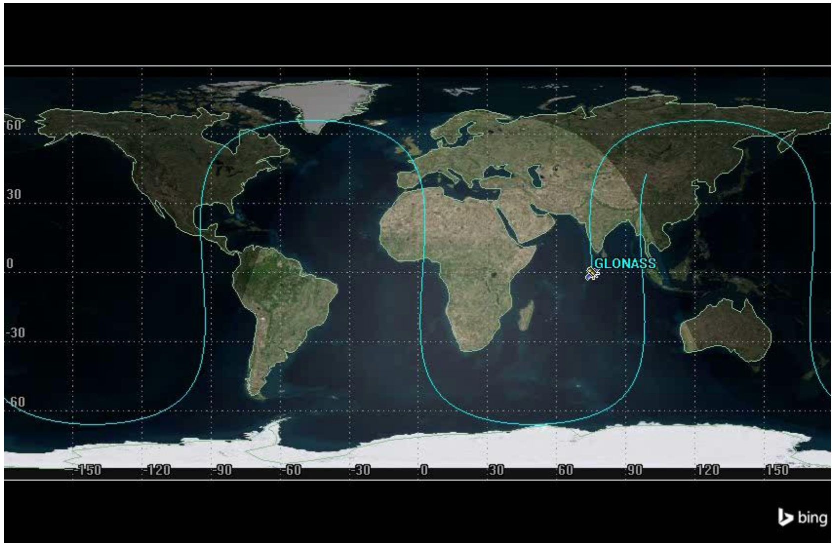
Medium Earth Orbit

T = 8/17
i = 66°
e = 0





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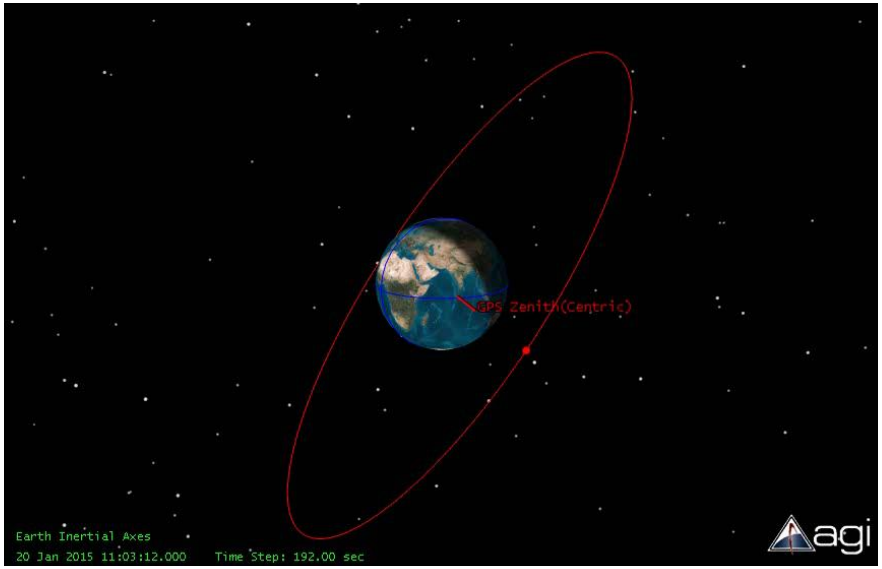
Medium Earth Orbit

T = 1/2
i = 55°
e = 0.02



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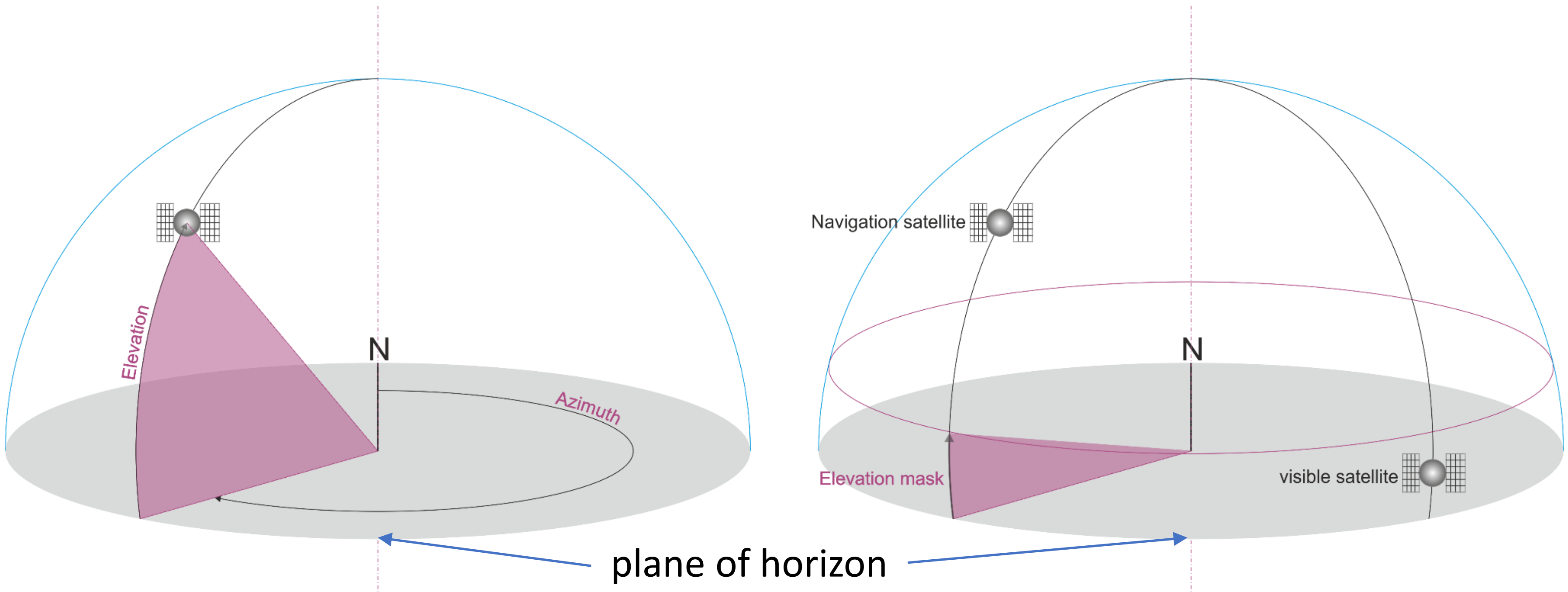
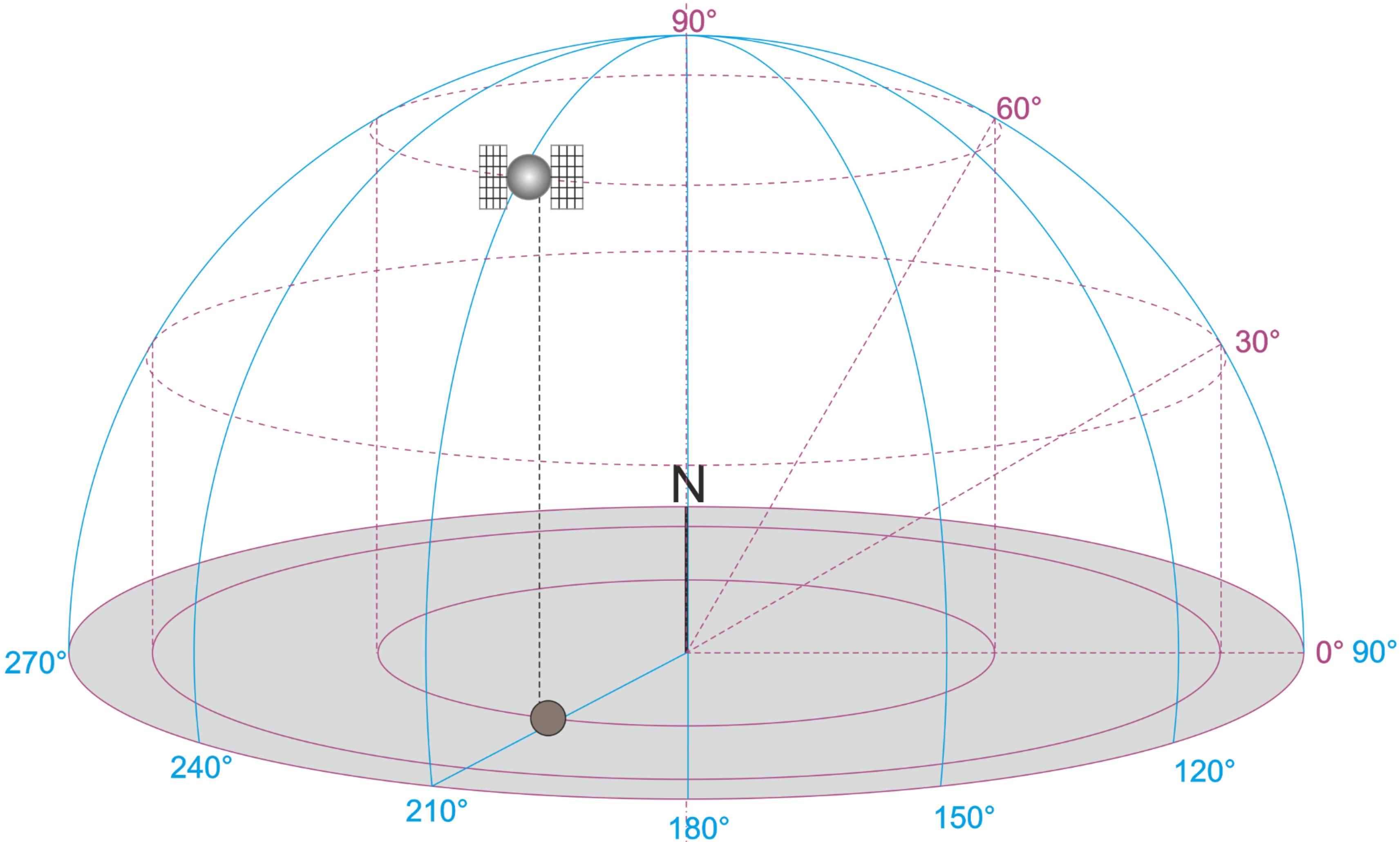
Position of satellite relative to observer

- Elevation
- Azimuth (bearing)
- Elevation mask
  - the minimum topocentric height of the satellite (elevation) of a navigation satellite
  - navigation satellite - which signal can be used to determine position of a receiver
  - visible satellite – above the plane of horizon



How to read a diagram „satellites” on EPFS



Az = 210°, El = 60°

The end