Navigation Satellite Systems
Principle of GPS operation

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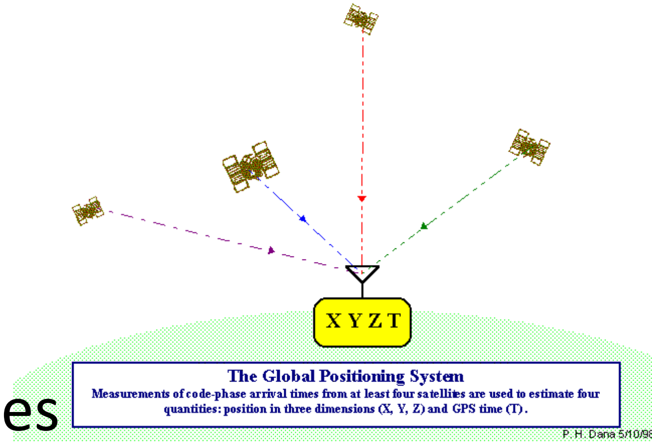
Positioning



- GPS provide 3D position:
  - φ, λ, h - geographical coordinates
  - x, y, z - ECEF (Earth Centered Earth Fix)
- The position is determined by measuring distances to GPS satellites
- The distance **d** is calculated basis on travel times of radio signals from satellite to receiver:

$$d = c \cdot \Delta t$$

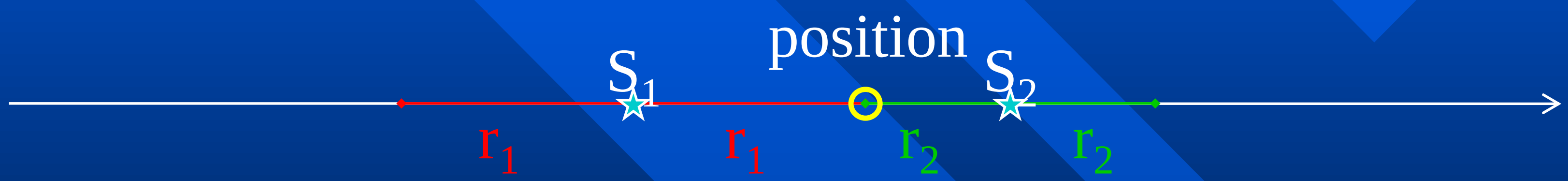
  - **c** – light speed**; Δt –** travel time (SV – R)

Pseudo-range

- Due to lack of synchronization of a receiver clock to GPS time, the distance is called pseudo-range
- The **pseudo-range** (navigation parameter) – the distance between SV and Receiver calculated basis on the travel time obtained by the receiver clock which is biased (offset) relative to GPS time

The distance as navigation parameter

- Position 1D
- Position 2D
- Position 3D



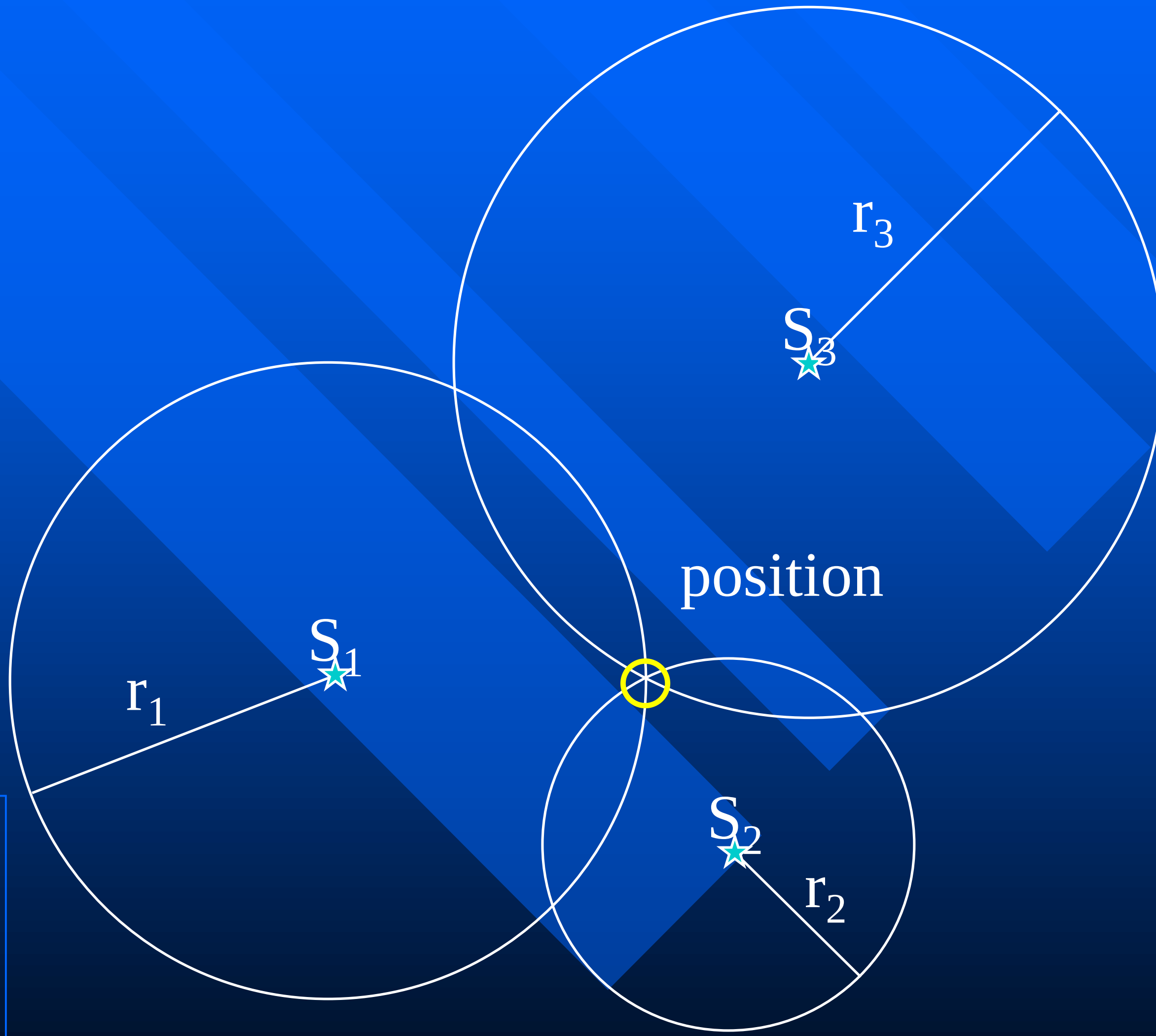
$$r_1 = \sqrt{(x - x_1)^2}$$

$$r_1^{\ 2} = (x - x_1)^2 \quad \Rightarrow \quad (x - x_1)^2 - r_1^{\ 2} = 0 \quad \Rightarrow \quad x^2 - 2x_1 x + x_1^{\ 2} - r_1^{\ 2} = 0$$

$$\begin{cases} r_1 = \sqrt{(x - x_1)^2} \\ r_2 = \sqrt{(x - x_2)^2} \end{cases}$$

The distance as navigation parameter

- Position 1D
- Position 2D
- Position 3D

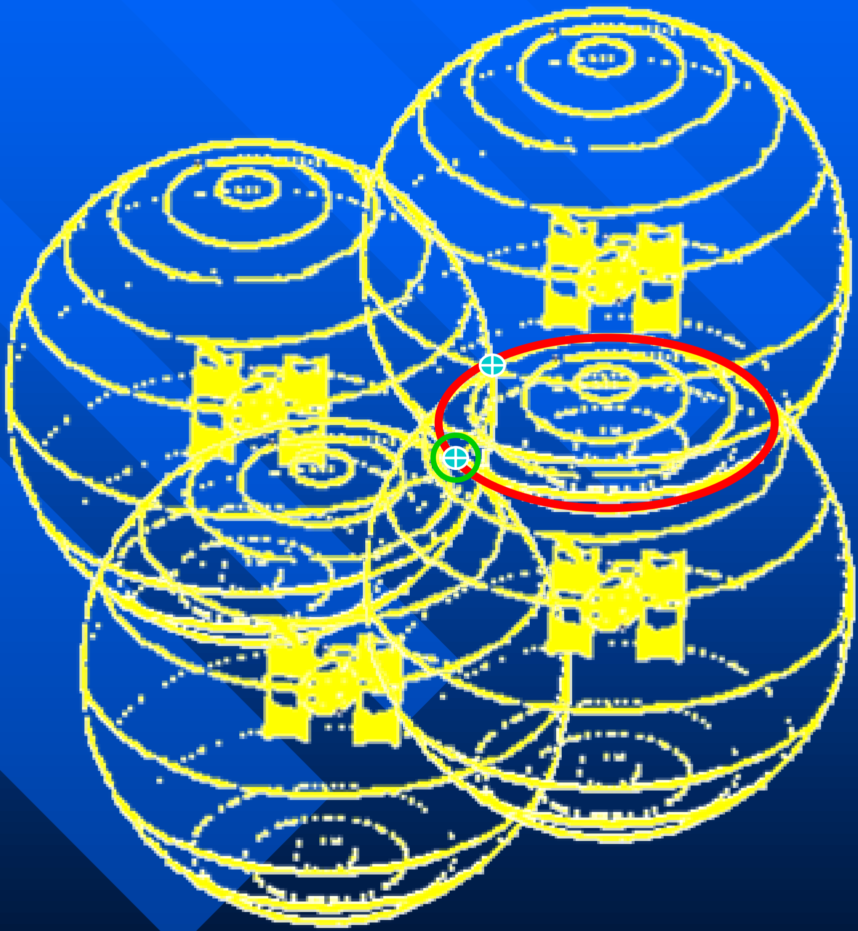


$$\begin{cases} r_1 = \sqrt{(x-x_1)^2 + (y-y_1)^2} \\ r_2 = \sqrt{(x-x_2)^2 + (y-y_2)^2} \\ r_3 = \sqrt{(x-x_3)^2 + (y-y_3)^2} \end{cases}$$

The distance as navigation parameter

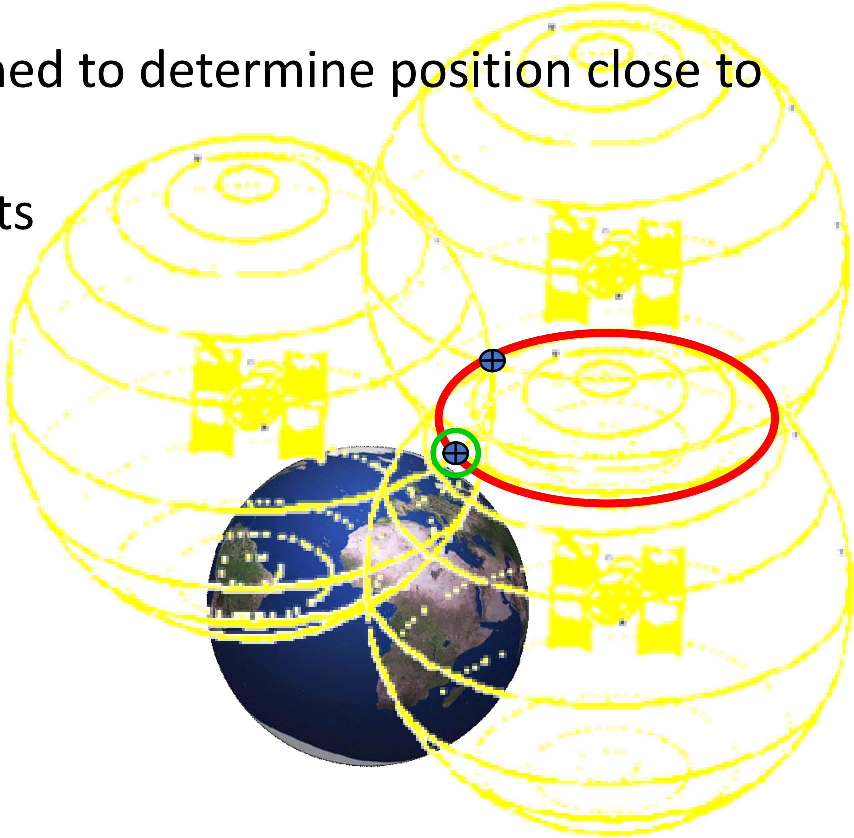
- Position 1D
- Position 2D
- Position 3D

$$
\begin{cases}
r_1 = \sqrt{(x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2} \\
r_2 = \sqrt{(x-x_2)^2 + (y-y_2)^2 + (z-z_2)^2} \\
r_3 = \sqrt{(x-x_3)^2 + (y-y_3)^2 + (z-z_3)^2} \\
r_4 = \sqrt{(x-x_4)^2 + (y-y_4)^2 + (z-z_4)^2}
\end{cases}
$$



The distance as navigation parameter / GPS

- GPS is a positioning system designed to determine position close to the Earth surface +/- 300 km
- With three distance measurements only one solution is within the positioning range, the second one is beyond this area and can be excluded
- but still, the fourth measurement is necessary to solve the receiver clock bias



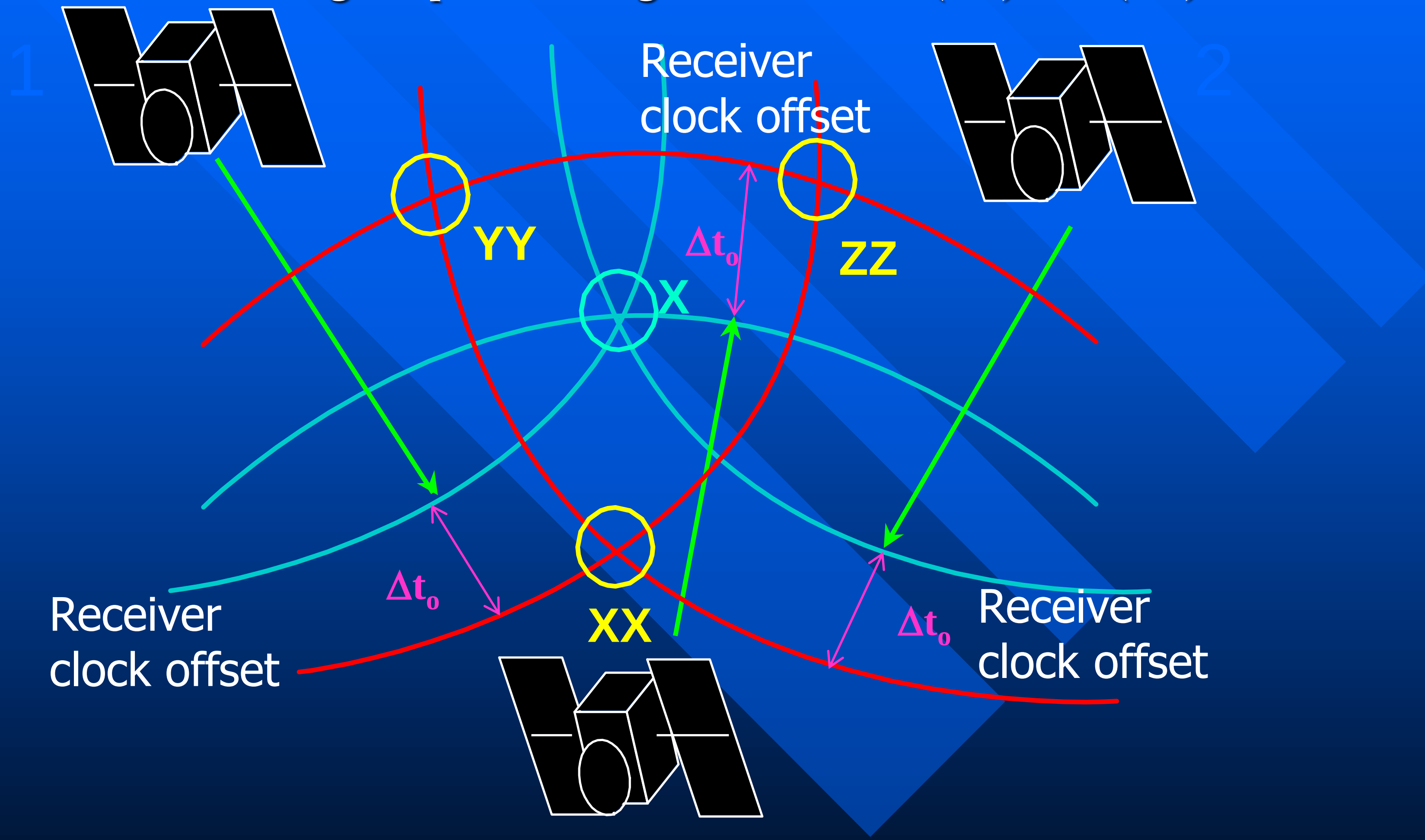
Travel time of satellite's signal

The travel time of SiS (signal in space) consist of:

- Δt_p – a propagation time between SV and R (distance, c)
- Δt_o – the receiver clock bias
- Δt_s – satellites' clocks biases
- Δt_i – signal delay in ionosphere
- Δt_t – signal delay in troposphere
- Δt_r – others like:
  - uneven rotation of the Earth
  - multipath
  - intentional interference
  - intentional reduction of accuracy (selective availability)

Determintion of receiver clock bias – Δt_o

Δt_o is treated as the fourth variable and calculated simultaneously on the basis of measuring the pseudo-ranges to at least 3 (2D) or 4 (3D) satellites



$$\boxed{c\Delta t_o = p_i - r_i \quad \Rightarrow \quad \mathrm{r_i} = p_i - c\Delta t_o}$$

The method of determining position

Taking into consideration the system of equations for position 3D and the receiver clock bias:

$$\begin{cases} \sqrt{(x-x_1)^2+(y-y_1)^2+(z-z_1)^2} = p_1 - \Delta t_o c \\ \sqrt{(x-x_2)^2+(y-y_2)^2+(z-z_2)^2} = p_2 - \Delta t_o c \\ \sqrt{(x-x_3)^2+(y-y_3)^2+(z-z_3)^2} = p_3 - \Delta t_o c \\ \sqrt{(x-x_4)^2+(y-y_4)^2+(z-z_4)^2} = p_4 - \Delta t_o c \end{cases}$$

Where:

(x, y, z) – receiver's coordinates (what we are looking for)

$\Delta t_o c$ – receiver clock bias expressed in meters

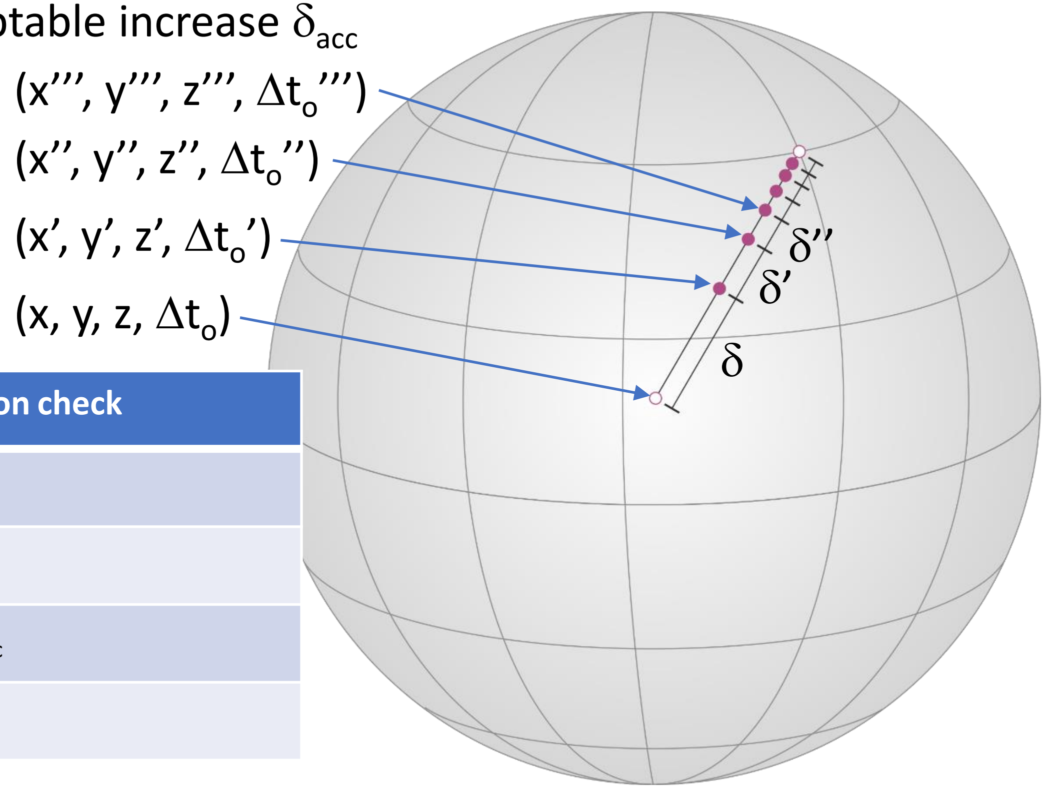
(x_i, y_i, z_i) – satellites' coordinates

p_i – pseudo-range

for i = 1..4

The method of determining position

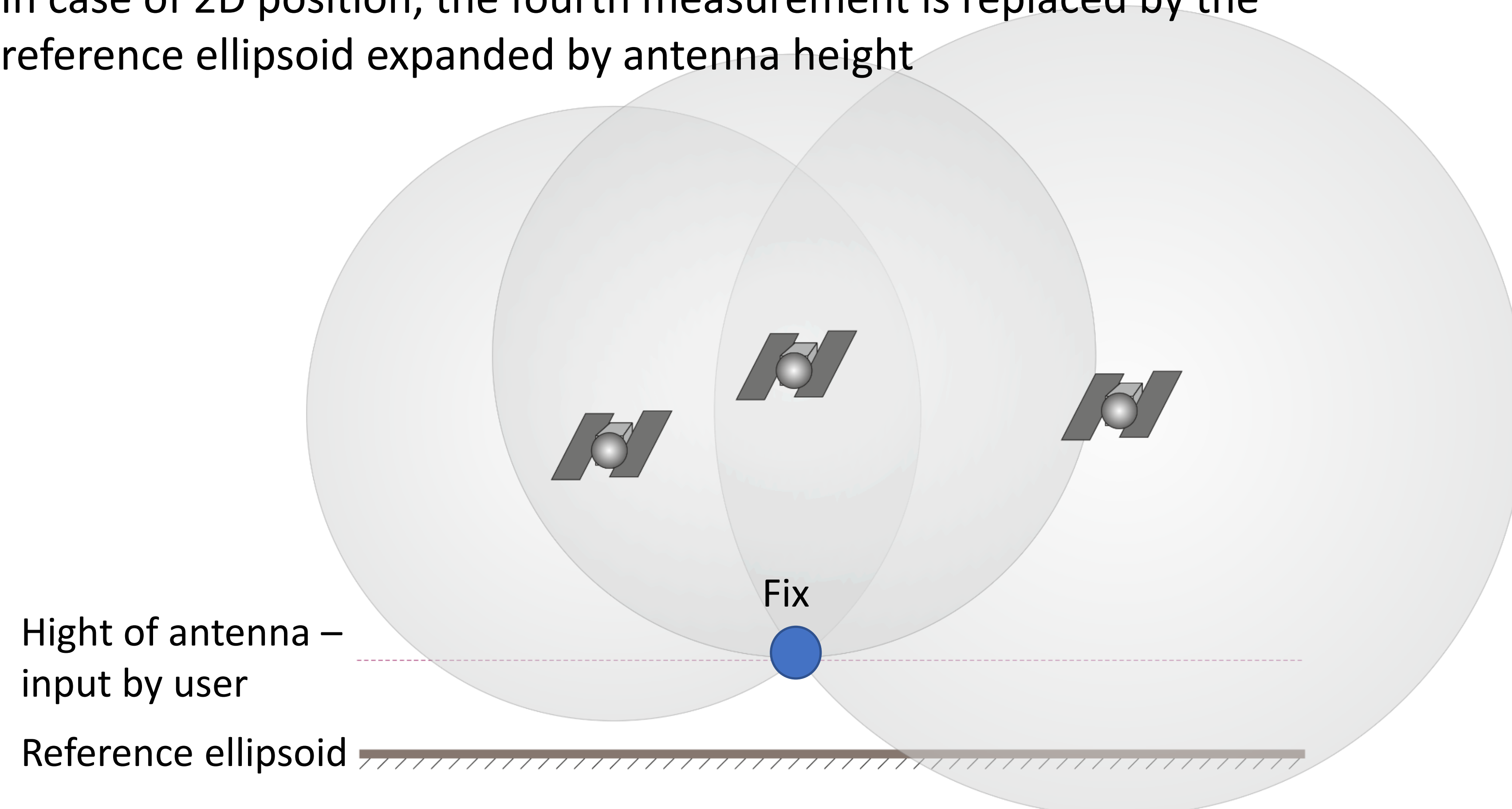
- To solve the position's system of equations a linearization and an iterations methods are used:
  - reducing the function to a linear form by develop it in Taylor’s expansion
  - Repetitions (iterations) of the positioning linear function till the increases of coordinates are below an acceptable increase δ_{acc}
  - δ_{acc} usually equals 0.5 m



No	Function (input)	Output	Condition check
1	f(x, y, z, Δt_o)	(x’, y’, z’, Δt_o’)	$\delta < \delta_{acc}$
2	f(x’, y’, z’, Δt_o’)	(x’’, y’’, z’’, Δt_o’’)	$\delta' < \delta_{acc}$
3	f(x’’, y’’, z’’, Δt_o’’)	(x’’’, y’’’, z’’’, Δt_o’’’)	$\delta'' < \delta_{acc}$
…	…	…	…

2D position vs 3D position

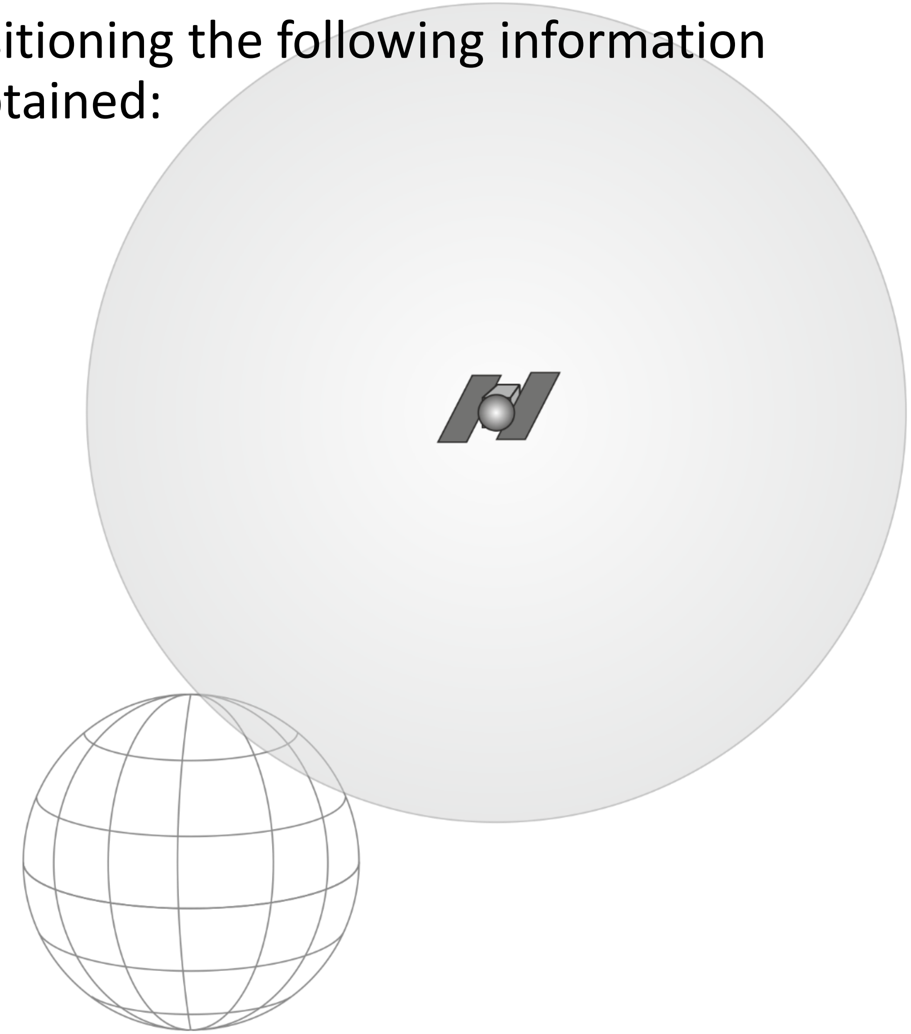
- In order to obtain 3D position the reception of signal from at least 4 satellites are required
- In case of 2D position, the fourth measurement is replaced by the reference ellipsoid expanded by antenna height



A surface of positioning

In order to establish a surface of positioning the following information from the satellite signal has to be obtained:

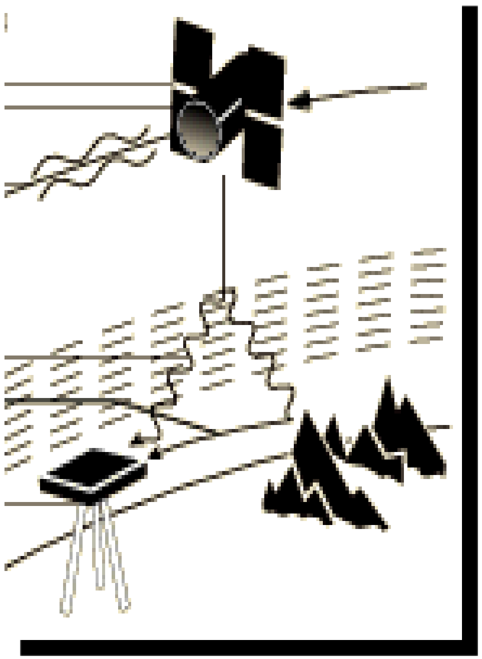
- When the signal has been sent;
  the travel time measurement
- Where was the satellite
  in a moment of sending the signal
- navigation message
  has to be received and decoded



Signals in space

SV transmits the following signals:

- S band – two channels for providing communication between the control segment and the space segment
- L band – two channels for users, consisting of satellites codes and the navigation message

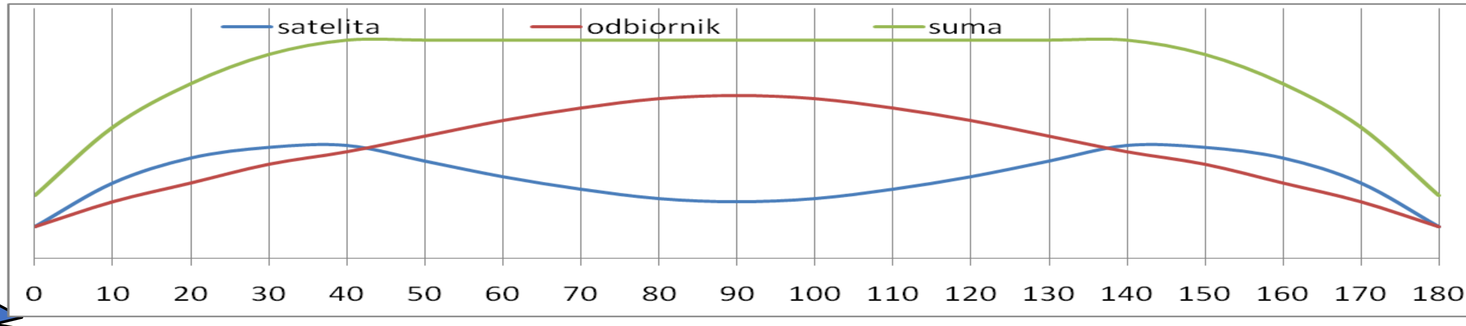


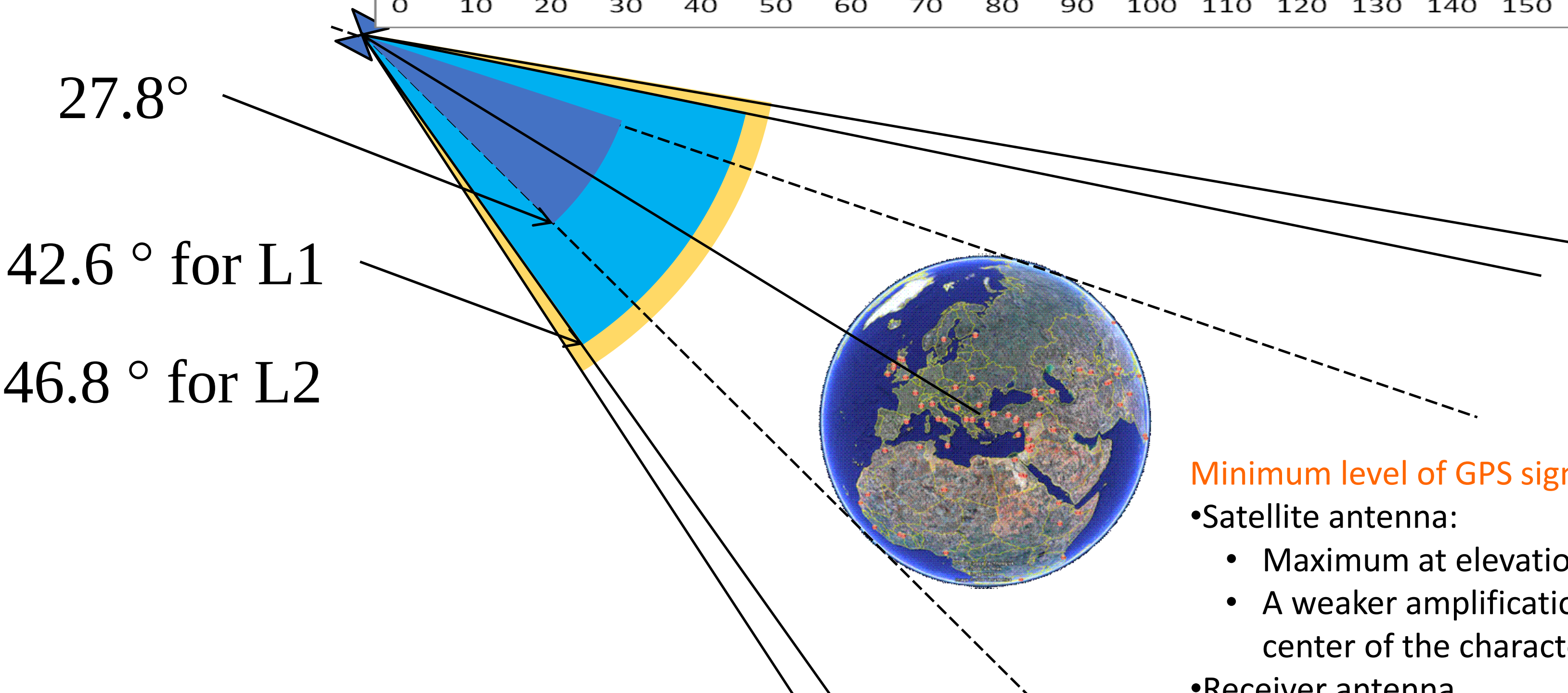
User signals

The carrier frequency:

- The base frequency: 10.23 Mhz
- two channels are created by multiplication of the base frequency
  - L1 = 154F = 1575.42 MHz
  - L2 = 120F = 1227.60 MHz
- due to a wave length, the possibility to measure distance with accuracy better then 0.2 m
- Receiving signals on two channels allowed to calculate the current corrections for an ionospheric delay
  - exploiting the dispersion phenomena
  - Better accuracy

Antenna characteristics



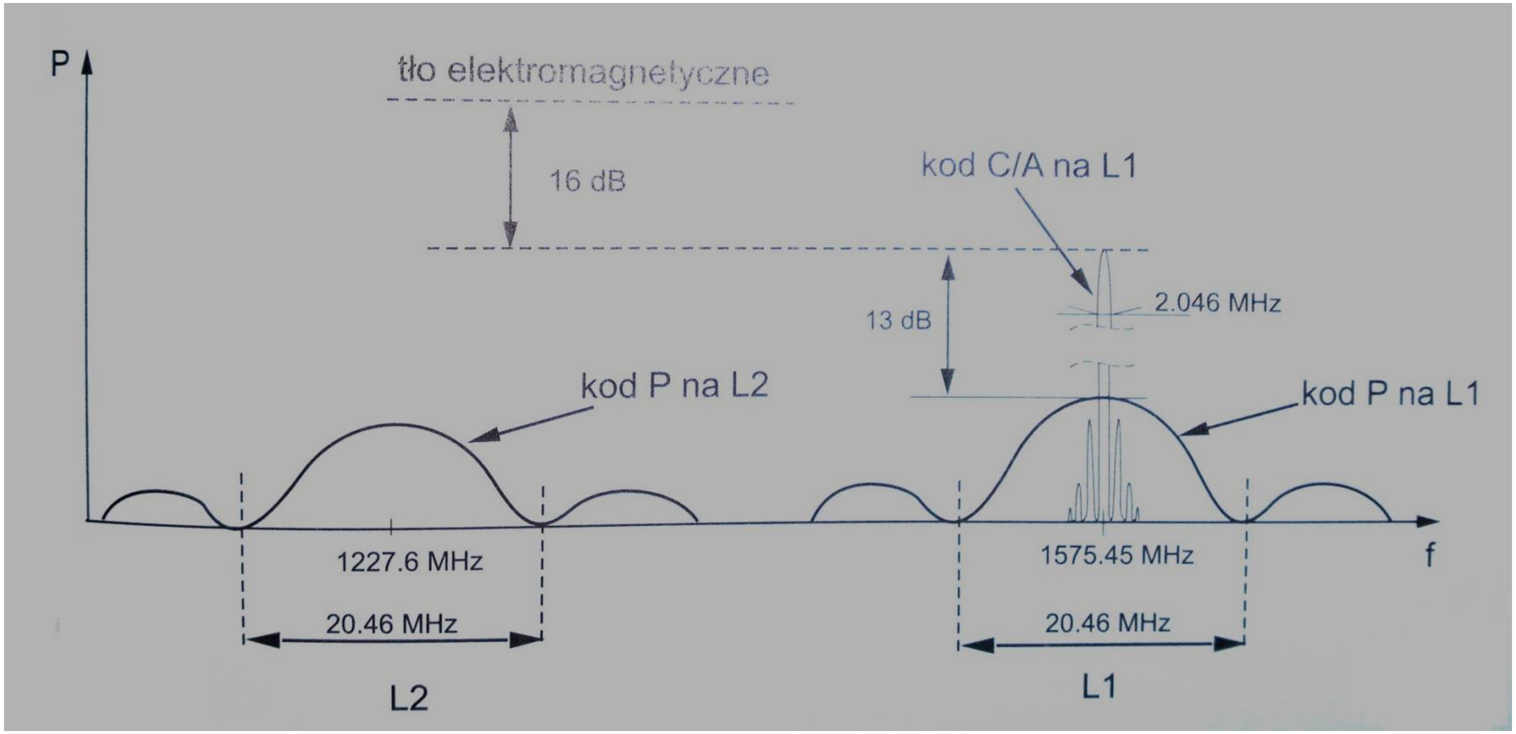


	P	C/A
L1	-163 dBm	-160 dBm
L2	-166 dBm	-166 dBm

Minimum level of GPS signals,

- Satellite antenna:
  - Maximum at elevation of 40°,
  - A weaker amplification in the center of the characteristic
- Receiver antenna
  - greater strengthening at the zenith (2.1 dBm)

The widthbands of transmissions



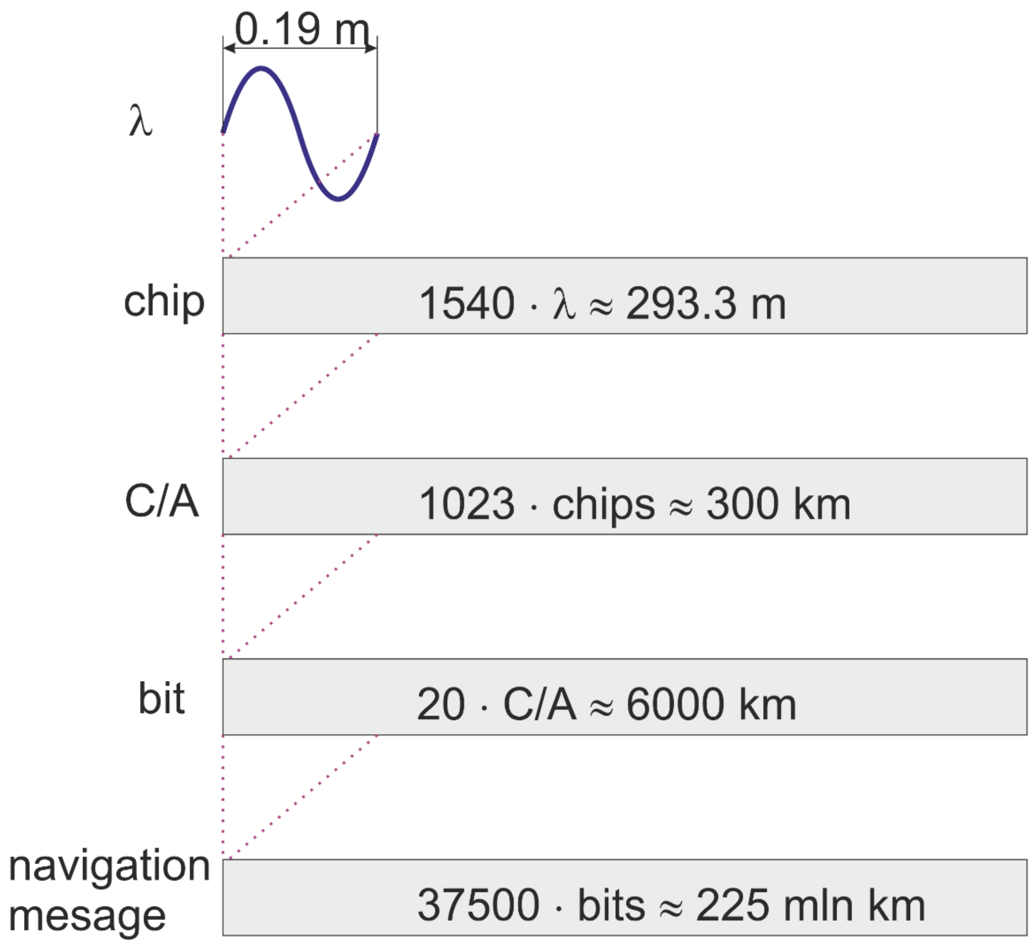
Width bands of particular signals:

- C/A ➔ 2.046 MHz
- P ➔ 20.46 MHz

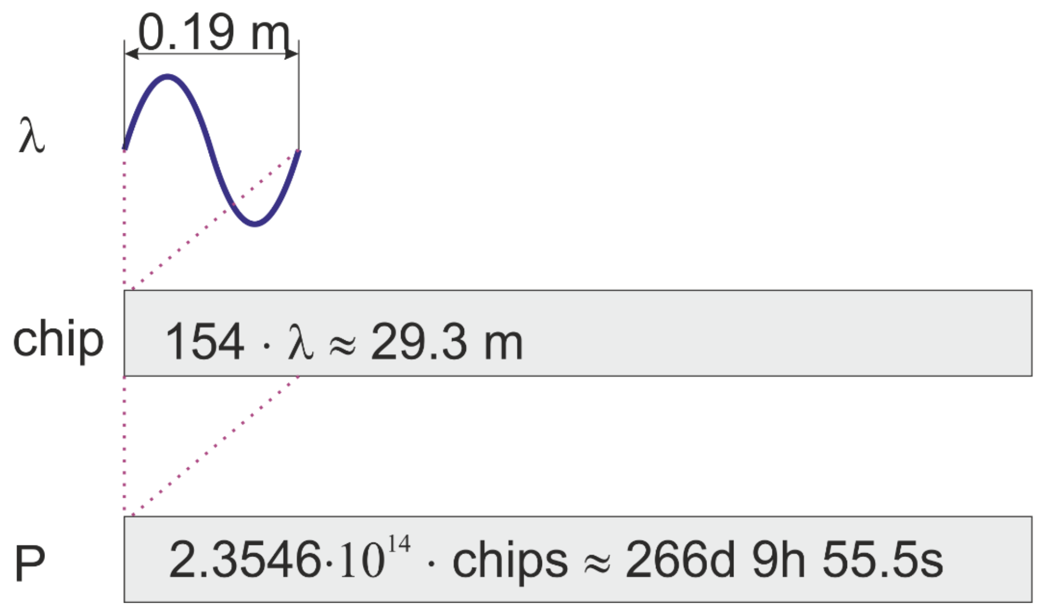
Signals structure

- L1
  - Code C/A
  - Code P
  - Navigation message
- L2
  - Code P
  - Navigation message

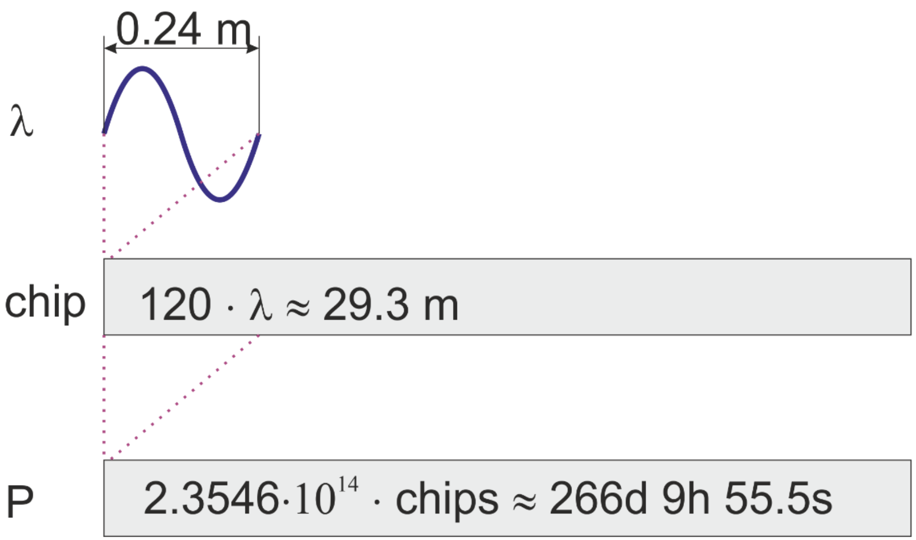
L1 C/A Signal structure

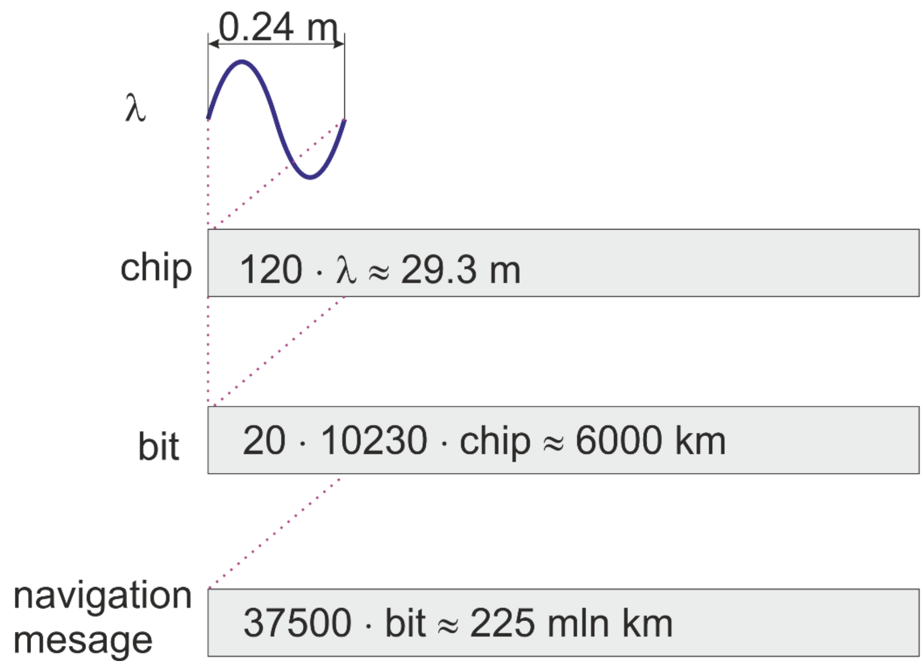


L1P Signal structure



L2P Signal structure



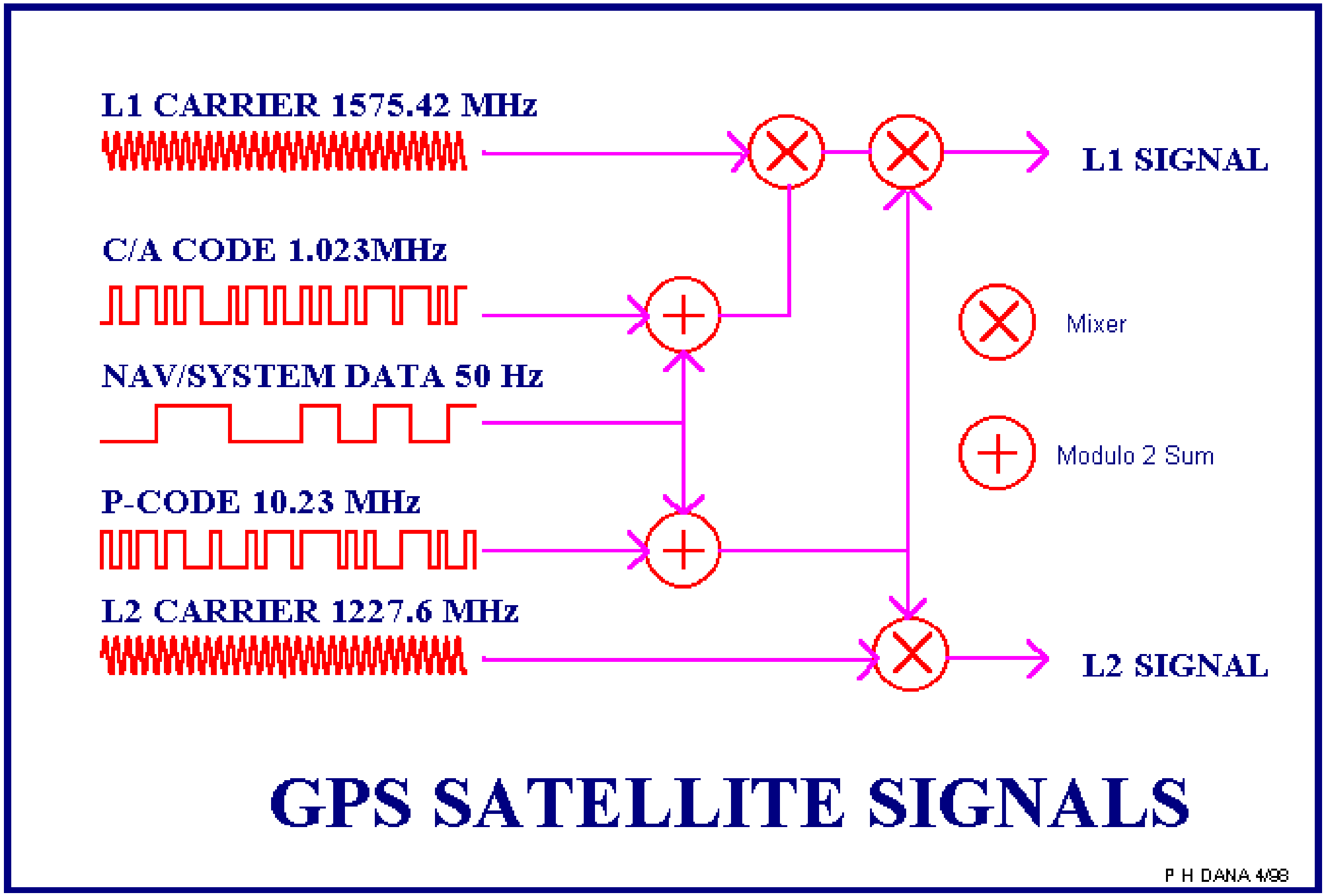


Signals structure

The signals transmitted by the GPS satellites are in the form of a carrier wave, phase-coded (BPSK - Binary Phase Shift Keying) by signals:

- Information (navigation message) with the clocked frequency 50 Hz
  50 b/s (bits per second)
- Pseudorandom code C/A with the clocked frequency 1.023 MHz
  1.023 Mb/s
- Pseudorandom code P with the clocked frequency 10.23 MHz
  10.23 Mb/s
- Pseudorandom code Y with the clocked frequency 0.5 Hz
  additional protection of P code => P(Y), anti-spoofing

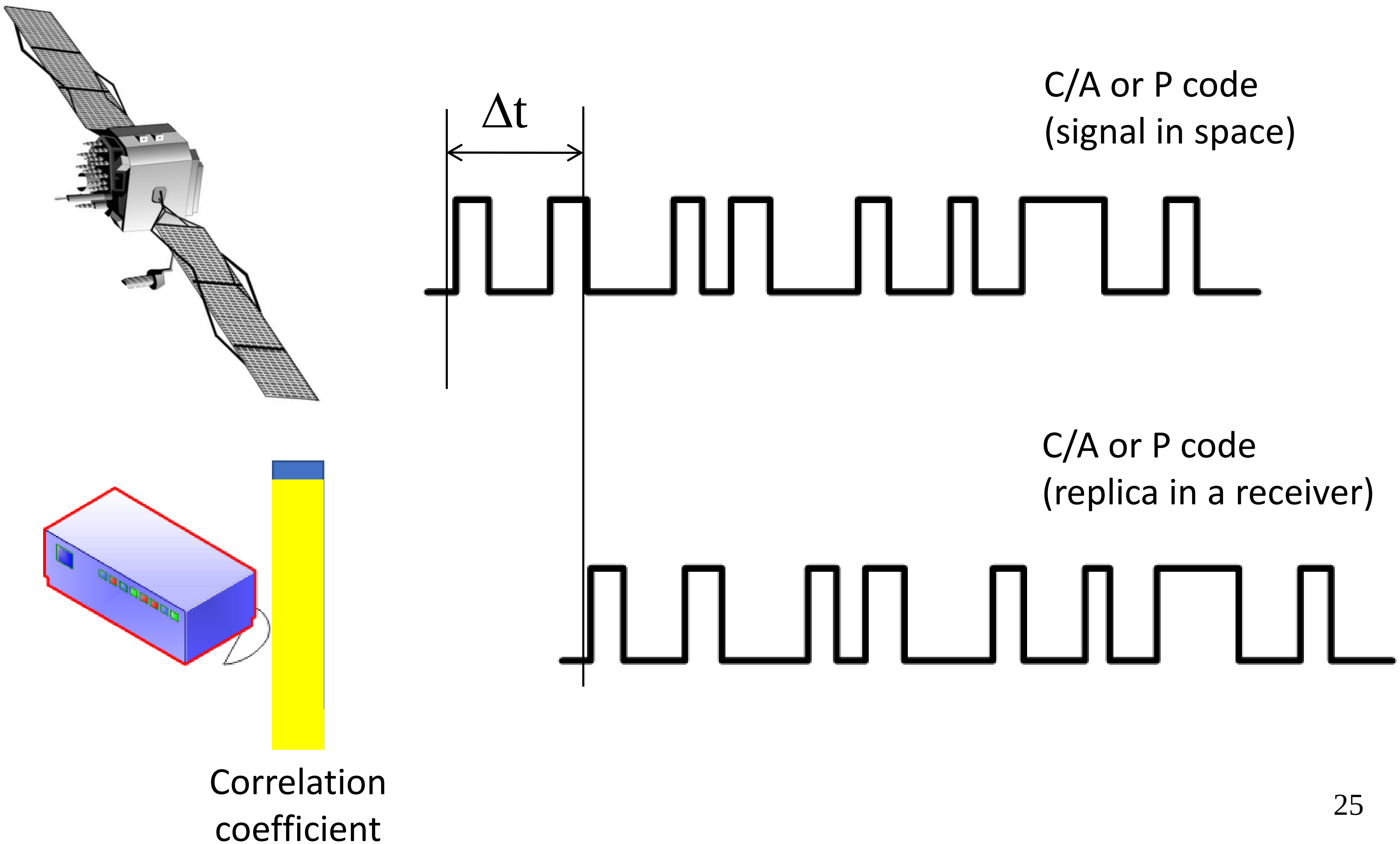
Signals in space



Signals transmitted by satellites – C/A, P

- The codes are generated as a Golda code
- Every satellite has its own section of such a pseudorandom code (C/A, P), which allow to identify the satellite
- The section of code, assigned to one SV are not correlated with any other sections
- Autocorrelation coefficient of the particular code section assigned for one satellite has only one maximum
- Signal travel time Δt is measured by shifting of a code replica generated in a receiver till achieve the maximum correlation coefficient with the code included into satellite signal.
- Travel time of satellite signals from a satellite to a receiver take 60 up to 81 ms

Travel time measurement Δt – code method



C/A code

- A public C/A code - a coarse acquisition; Transmitted only on L1
- Each satellite has a different code (different section of a Golda code)
- Code bits are called chips (1 μs - 300 m)
- Modulated sections by the pseudorandom code with a length of 1023 bits
- Transmission speed 1.023 Mbit / s
- Repeatability of the code of a given satellite - 1ms which enables fast synchronization of the code generated in the receiver with the included one in transmitted satellite signal
- 20 sections of code per 1 bit of a navigation message
- Measurement of the time of reaching the receiver with the resolution of 1% of a chip, which means: 0.01 μs at the light speed of 300,000 km/s - enables measurement accuracy up to 3m
- SPS – standard positioning serwis

P code

- The P - precise - exact code available only for authorized users
- Broadcast on both channels L1, L2
- Modulated in sections of one week, as a portion of lasting more than 267 days pseudorandom Golda code
- SV is assigned a one-week segment of this code
- Resumption of the sequence repetition takes place on Saturdays after midnight.
- Transmission speed - 10.23 MHz (10 * C / A)
- Increased ten times the accuracy of pseudo-range measurement (0.30 m)

Y code

- Transmitted in special situations - to prevent interfere of GPS signals in space (anti-spoofing)and secure a proper operation of receivers
- During such a situation P code is modulated by Y code (0.5 Hz) in order to secure PPS service (Precise Positioning Service)

Conclusions

The assumptions of the system work in relation to:

- position accuracy - 10m,
- speed determining accuracy 0.1 m/s,
- time transfer accuracy 1 μs.

have been fulfilled

Navigation message

- Available on L1 and L2
- The data includes information required to determine the following:
  - Satellite time-of-transmission
  - Satellite position
  - Satellite health
  - Satellite clock correction
  - Ionospheric delay effects
  - Time transfer to Coordinated Universal Time as kept by the U.S. Naval Observatory [UTC(USNO)]
  - Constellation status
- Transfer speed 50 b/s
- Consists of 25 frames with 1500 bits each
- Each frame is divided into 5 subframes with 300 bits each, 10 words with 30 bits each
- Receiving one frame takes 30 s, and whole message (25 frame) 12.5 minutes

Navigation message

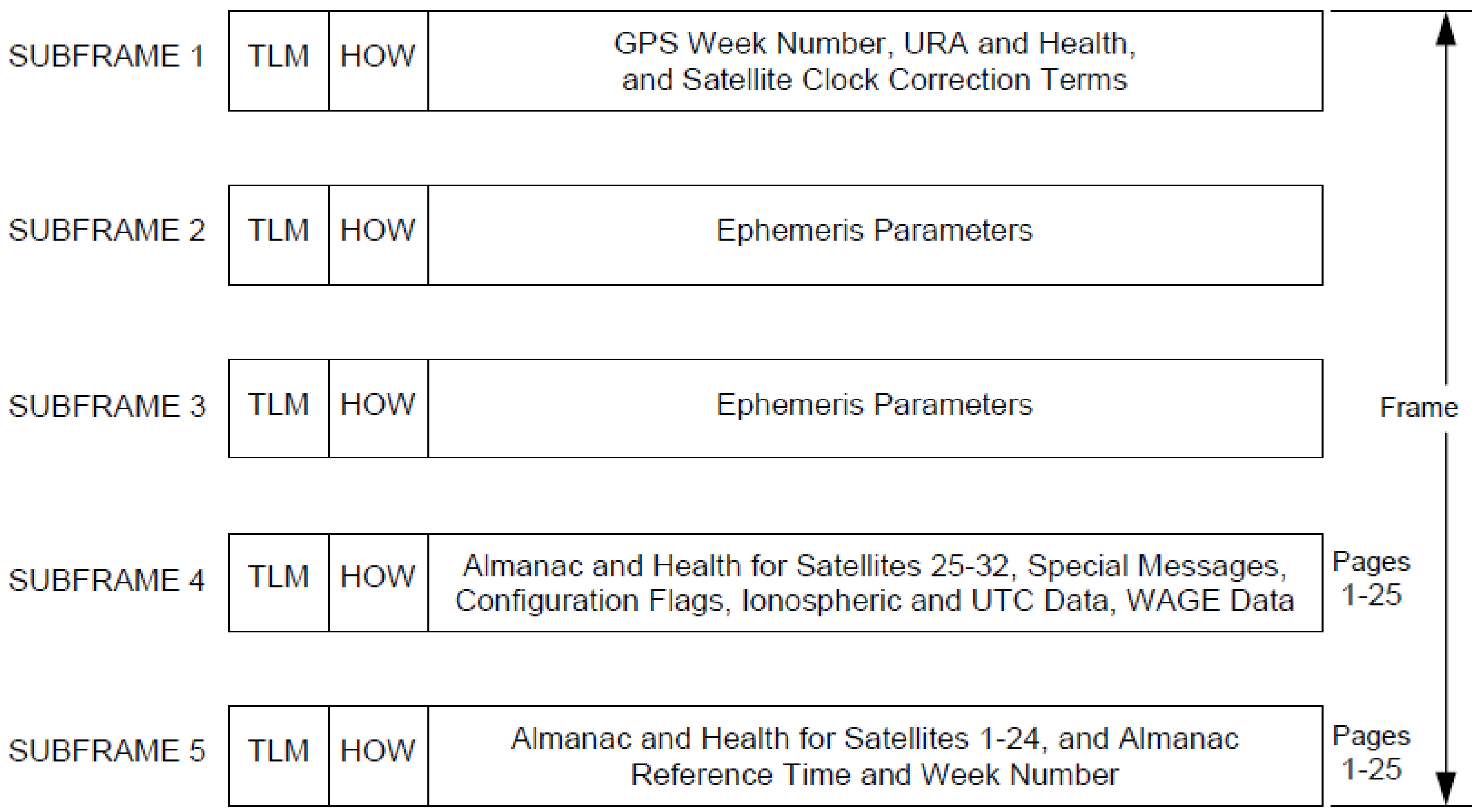
- Subframes 1,2 and 3 repeating the same 900 bits of data in all 25 frames, which allow to get critical information within 30 seconds
- The critical data are updated every 1-2 hours
- Whole navigational message is updated every 4 hours

One frame consists of:

- 1,2,3 subframes – orbit parameters of a given satellite (the critical data)
- 4,5 subframes – information to determine
  - UTC time
  - Correction of ionospheric delay
  - Almanac of the GPS constellation

NAV Message Content and Format Overview

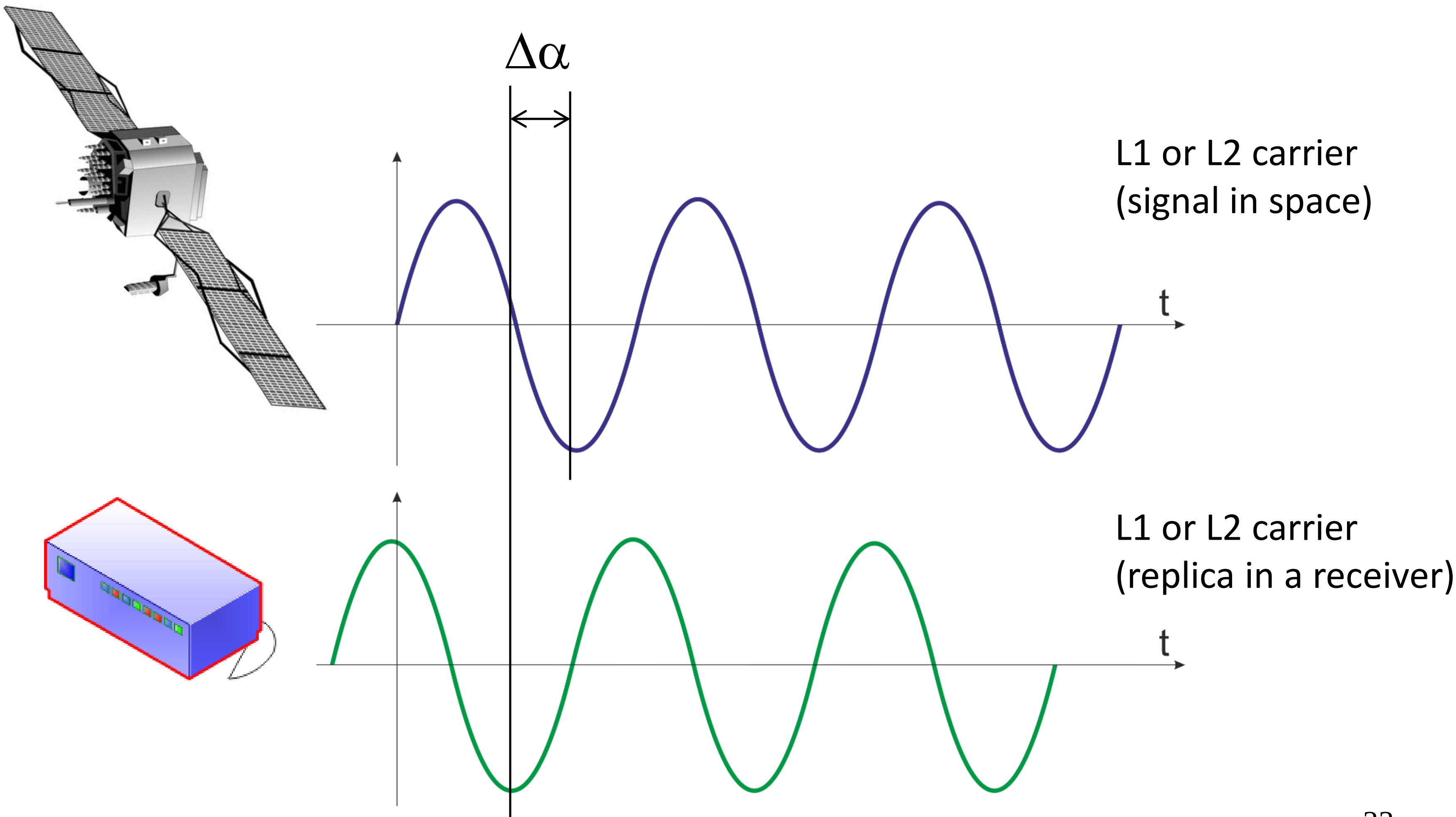
Significant Subframe Contents



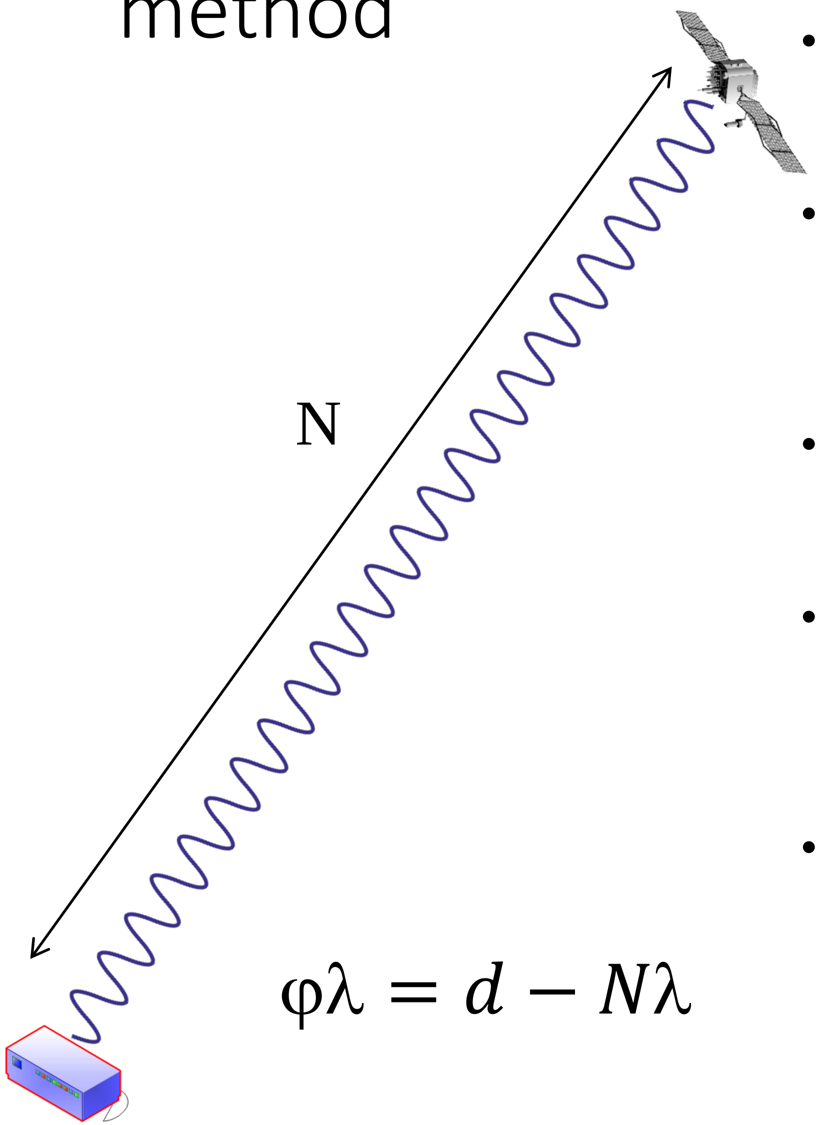
Navigation message

- Subframe begins with a word
  - TLM – telemetric information – which allows to identify SV and synchronize pseudorandom codes generated in receivers with these in signal in space (from a given satellite)
  - HOW – hand over word – which allows to fine tuning receiver's signal to satellite's signal, recognize pseudorandom codes and overpass to receiving code P (authorized receivers)
- Pseudorandom codes are restarted at Saturday's midnight every week.

Travel time measurement $\Delta\alpha$ – phase method



Travel time measurement $\Delta\alpha$ – phase method



$$\varphi\lambda = d - N\lambda$$

- N – integer number of full lengths of electromagnetic wave (uncertainty of phase measurement)
- This number can be interpreted as a difference between a value of an initial integrated phase and a distance to a given satellite. The uncertainty of phase is varied to every satellite
- The uncertainty N remains constant during measurements, if the satellite signal tracking process is not disrupted
- The occurrence of such a disturbance (i.e. multipath) results in a phase discontinuity (Cycle Slip), i.e. a step change in the registered phase by the total number of cycles (wave lengths)
- Detection and correction of the phase discontinuity is critical for precise geodetic measurements using phase observations, in particular RTK techniques (real time kinematic)

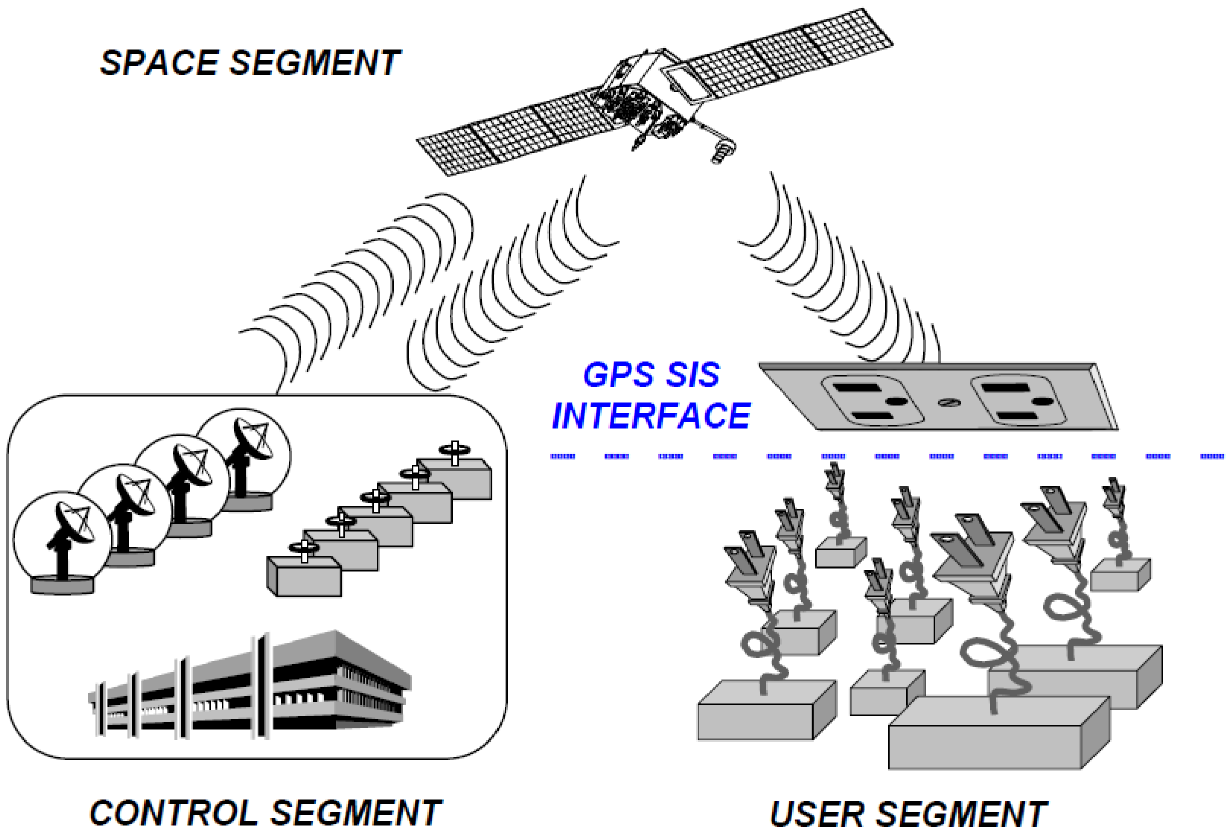
Positioning services

- SPS – Standard Positioning Service – standard accuracy, available to all users. The error budget consists of:
  - SA – selective availability – intentionally, by system's owner, degraded satellite signals. This procedure discontinued in 2000
  - Satellite's clock error
  - Ionospheric delay (value determined using model of ionosphere)
  - Tropospheric delay
  - Receiver noise and resolution
  - Multipath
  - Other
- PPS – Precise Positioning Service – improved accuracy, available to authorized users. The error budget consists of:
  - Satellite's clock error
  - Ionospheric delay (current value determined using a dispersion phenomena)
  - Tropospheric delay
  - Receiver noise and resolution
  - Multipath
  - Other

GPS accuracy

Currently GPS accuracy is described by signal in space (SiS) parameters. Therefore from **the organizational perspective** GPS system has only two components:

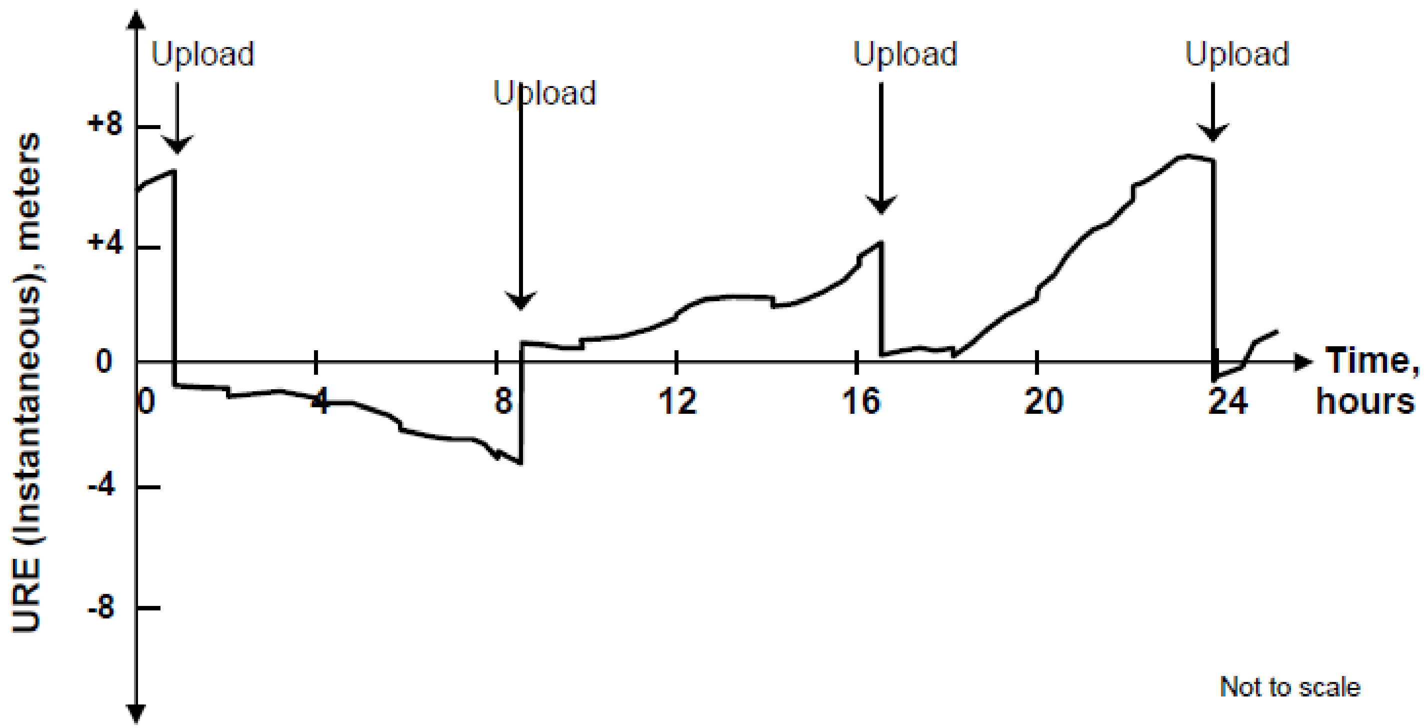
- Space segment
- Control segment



L1 Single-Frequency C/A-Code UERE Budget

Segment	Error Source	UERE Contribution (95%) (meters)		
		Zero AOD	Max. AOD in Normal Operation	14.5 Day AOD
Space	Clock Stability	0.0	8.9	257
	Group Delay Stability	3.1	3.1	3.1
	Diff'l Group Delay Stability	0.0	0.0	0.0
	Satellite Acceleration Uncertainty	0.0	2.0	204
	Other Space Segment Errors	1.0	1.0	1.0
Control	Clock/Ephemeris Estimation	2.0	2.0	2.0
	Clock/Ephemeris Prediction	0.0	6.7	206
	Clock/Ephemeris Curve Fit	0.8	0.8	1.2
	Iono Delay Model Terms	9.8-19.6	9.8-19.6	9.8-19.6
	Group Delay Time Correction	4.5	4.5	4.5
	Other Control Segment Errors	1.0	1.0	1.0
User*	Ionospheric Delay Compensation	N/A	N/A	N/A
	Tropospheric Delay Compensation	3.9	3.9	3.9
	Receiver Noise and Resolution	2.9	2.9	2.9
	Multipath	2.4	2.4	2.4
	Other User Segment Errors	1.0	1.0	1.0
95% System UERE (SPS)		12.7-21.2	17.0-24.1	388
* For illustration only, actual SPS receiver performance varies significantly -- see Table B.2-1				

Instantaneous SPS SIS URE



Dual-Frequency P(Y)-Code UERE Budget

Segment	Error Source	UERE Contribution (95%) w/o WAGE (meters)		
		Zero AOD	Max. AOD in Normal Operation	14.5 Day AOD
Space	Clock Stability	0.0	8.9	257
	Group Delay Stability	0.0	0.6	0.6
	Diff'l Group Delay Stability	0.0	2.0	2.0
	Satellite Acceleration Uncertainty	0.0	2.0	204
	Other Space Segment Errors	1.0	1.0	1.0
Control	Clock/Ephemeris Estimation	2.0	2.0	2.0
	Clock/Ephemeris Prediction	0.0	6.7	206
	Clock/Ephemeris Curve Fit	0.8	0.8	1.2
	Iono Delay Model Terms	N/A	N/A	N/A
	Group Delay Time Correction	N/A	N/A	N/A
	Other Control Segment Errors	1.0	1.0	1.0
User*	Ionospheric Delay Compensation	4.5	4.5	4.5
	Tropospheric Delay Compensation	3.9	3.9	3.9
	Receiver Noise and Resolution	2.9	2.9	2.9
	Multipath	2.4	2.4	2.4
	Other User Segment Errors	1.0	1.0	1.0
95% System UERE (PPS)		7.5	13.8	388

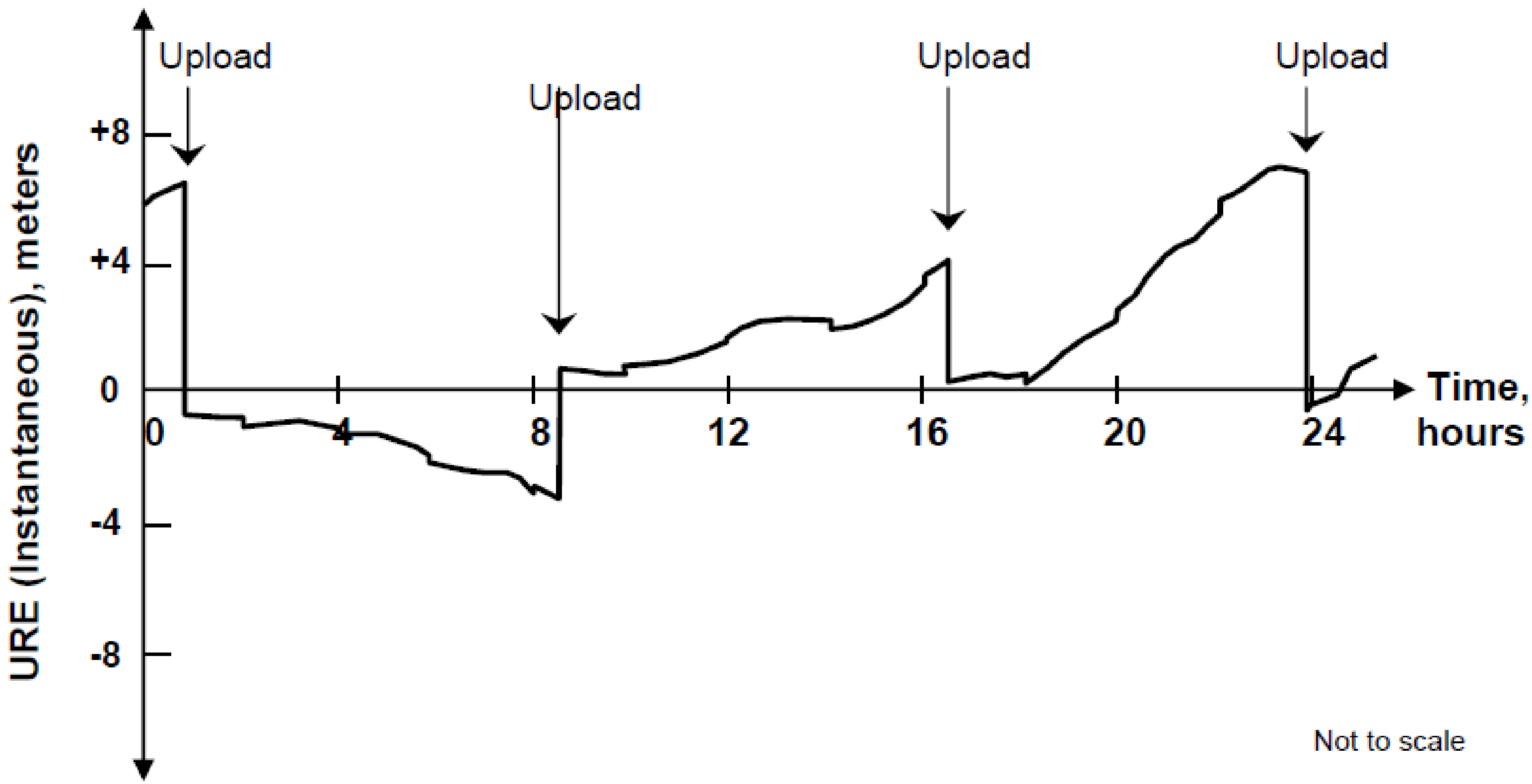
* For illustration only, actual PPS receiver performance varies significantly -- see Table B.2-1

L1 Single-Frequency P(Y)-Code UERE Budget

Segment	Error Source	UERE Contribution (95%) w/o WAGE (meters)		
		Zero AOD	Max. AOD in Normal Operation	14.5 Day AOD
Space	Clock Stability	0.0	8.9	257
	Group Delay Stability	1.6	1.6	1.6
	Diff'l Group Delay Stability	0.0	0.0	0.0
	Satellite Acceleration Uncertainty	0.0	2.0	204
	Other Space Segment Errors	1.0	1.0	1.0
Control	Clock/Ephemeris Estimation	2.0	2.0	2.0
	Clock/Ephemeris Prediction	0.0	6.7	206
	Clock/Ephemeris Curve Fit	0.8	0.8	1.2
	Iono Delay Model Terms	9.8-19.6	9.8-19.6	9.8-19.6
	Group Delay Time Correction	4.5	4.5	4.5
	Other Control Segment Errors	1.0	1.0	1.0
User*	Ionospheric Delay Compensation	N/A	N/A	N/A
	Tropospheric Delay Compensation	3.9	3.9	3.9
	Receiver Noise and Resolution	2.9	2.9	2.9
	Multipath	2.4	2.4	2.4
	Other User Segment Errors	1.0	1.0	1.0
95% System UERE (PPS)		12.5-21.1	16.8-23.9	388

* For illustration only, actual PPS receiver performance varies significantly -- see Table B.2-1

Instantaneous PPS SIS URE



Basic Equations for PVT Accuracy

The basic equation for PVT (Position, Velocity, and Time) accuracy is:

- Accuracy = UERE x DOP
  - UERE – user estimate range error

- UHNE = User Horizontal Navigation Error (rms)
  - UHNE = UERE x HDOP
- UVNE = User Vertical Navigation Error (rms)
  - UVNE = UERE x VDOP
- UHVE = User Horizontal Velocity Error (rms)
  - UHVE = URRE x HDOP
- UVVE = User Vertical Velocity Error (rms)
  - UVVE = URRE x VDOP
- UTE = User Time Error (rms)
  - UTE = UERE x TDOP / c

URRE – User Range Rate Error

SPS PS Global-Average Position Accuracy Standards

- 13 m 95% Horizontal Error
- 22 m 95% Vertical Error

The end